

Supplement

Power Computations for Intervention Analysis

The result of Tiao et al. (1990) is shown to be a special case of our results in Table 1 for a ramp intervention with AR(1) noise.

Tiao et a. (1990, Appendix A), p.20,515

Let $\mathcal{R}_t^{(T)}$ denote the ramp intervention defined in Tiao (1990) so we have $\mathcal{R}_t^{(T)} = R_t^{(T+1)} / 12$. Then assuming $T = 0$ the following expression was derived for $\sigma_{\hat{\omega}}$,

$$\sigma_{\hat{\omega}}^{(\text{Tiao})} = \sigma g(T, \phi),$$

where $\sigma = \sigma_a^2 / (1 - \phi^2)$ and $g(T, \phi) = 24 (3 (1 + \phi) / ((1 - \phi) T (T^2 - 1)))^{1/2}$

In *Mathematica* this expression can be defined as:

```
24 (3 (1 + φ) / ((1 - φ) T (T^2 - 1)))1/2 // Simplify
```

$$24 \sqrt{3} \sqrt{-\frac{1 + \phi}{(-T + T^3) (-1 + \phi)}}$$

```
gTiao = %
```

$$24 \sqrt{3} \sqrt{-\frac{1 + \phi}{(-T + T^3) (-1 + \phi)}}$$

Comparison With Our Result

Let $\sigma_{\hat{\omega}}(T, \phi)$ denote our estimate. Then taking into account $\mathcal{R}_t^{(T)} = R_t^{(T+1)} / 12$. We obtain, $g(T, \phi) = \sqrt{(1 - \phi^2) / 12} \sigma_{\hat{\omega}}(T + 1, \phi)$ as the corresponding value in our notation. In order to compare with our result we must assume that the mean is known.

■ Definition of *Mathematica* Function

The results in our Table 1 may be written as:

```

InformationMatrixAR1Ramp[n_, T_, ϕ_] :=
  With[{κ = ϕ - 1,
        b12 = (1 - ϕ) (n + 1 - T) (n + 2 - T - ϕ (n - T)) / 2,
        b22 =  $\frac{1}{6} (1 + n - T) (6 + 7 n + 2 n^2 - 7 T - 4 n T + 2 T^2 - 8 n \phi - 4 n^2 \phi + 8 T \phi + 8 n T \phi - 4 T^2 \phi + n \phi^2 + 2 n^2 \phi^2 - T \phi^2 - 4 n T \phi^2 + 2 T^2 \phi^2)$ }, {n κ2, b12}, {b12, b22}}
]

```

■ Special Case

The special case corresponds to mean known so,

```

g[T_, ϕ_] :=
  (Sqrt[Inverse[InformationMatrixAR1Ramp[T, 1, ϕ]] // Flatten // Last]) * Sqrt[1 - ϕ2] * 12

g[T, ϕ] // FullSimplify

```

$$24 \sqrt{\frac{1}{(-T + T^3) (-1 + \phi)^2}} \sqrt{3 - 3 \phi^2}$$

■ Proof of Equivalence

Both expressions are positive, so equality of the squared result establishes the equivalence.

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gTiao2 - g[T, ϕ]2 // FullSimplify

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0