Supplement

Power Computations for Intervention Analysis

The result of Tiao et al. (1990) is shown to be a special case of our results in Table 1 for a ramp intervention with AR(1) noise.

Tiao et a. (1990, Appendix A), p.20,515

Let \( h_t \) denote the ramp intervention defined in Tiao (1990) so we have

\[
R_t = h_t + \frac{1}{\sqrt{12}}.
\]

Then assuming \( T = 0 \) the following expression was derived for \( \sigma_\omega \),

\[
\sigma_\omega^{(\text{Tiao})} = \sigma g(T, \phi),
\]

where \( \sigma = \sigma_\omega^2 / (1 - \phi^2) \) and \( g(T, \phi) = 24 (3 + \phi) / ((1 - \phi) T(T^2 - 1))^1/2 \)

In Mathematica this expression can be defined as:

\[
24 \sqrt{3} \left( \frac{1 + \phi}{(-T + T^3)(-1 + \phi)} \right)^{1/2} // \text{Simplify}
\]

\[
g_{\text{Tiao}} = \%
\]

Comparison With Our Result

Let \( \sigma_{\omega}(T, \phi) \) denote our estimate. Then taking into account \( R_t^{(T)} = R_{t+1}^{(T)} / 12 \). We obtain,

\[
g(T, \phi) = \sqrt{(1 - \phi^2) / 12} \cdot \sigma_{\omega}(T + 1, \phi)
\]

as the corresponding value in our notation. In order to compare with our result we must assume that the mean is known.

- Definition of Mathematica Function

The results in our Table 1 may be written as:
InformationMatrixAR1Ramp[n_, T_, φ_] := 
With[{κ = φ - 1, 
    b12 = (1 - φ) (n + 1 - T) (n + 2 - T - φ (n - T))/2, 
    b22 = 1/6 (1 + n - T) (6 + 7 n + 2 n^2 - 7 T - 4 n T + 2 T^2 - 8 n φ - 4 n^2 φ + 8 T φ + 8 n T φ - 
        4 T^2 φ + n φ^2 + 2 n^2 φ^2 - T φ^2 - 4 n T φ^2 + 2 T^2 φ^2)}, {{n κ^2, b12}, {b12, b22}}]

■ Special Case

The special case corresponds to mean known so,

\[ g_{\text{special}}(T, \phi) = \left( \text{Sqrt[Inverse[InformationMatrixAR1Ramp[T, 1, \phi]] // Flatten // Last]} \right) \times \text{Sqrt[1 - \phi^2]} \times 12 \]

\[ g[T, \phi] // \text{FullSimplify} \]

\[ 24 \sqrt{\frac{1}{(-T + T^3) (-1 + \phi)^2}} \sqrt{3 - 3 \phi^2} \]

■ Proof of Equivalence

Both expressions are positive, so equality of the squared result establishes the equivalence.

\[ g_{\text{Tiao}}^2 - g[T, \phi]^2 // \text{FullSimplify} \]

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