STAT 2857 — Homework 1

Exercise 1. *A survey of a group's viewing habits over the last year revealed the following information:*

(i) *28% watched gymnastics*

(ii) *29% watched baseball*

(iii) *19% watched soccer*

(iv) *14% watched gymnastics and baseball*

(v) *12% watched baseball and soccer*

(vi) *10% watched gymnastics and soccer*

(vii) *8% watched all three sports.*

Calculate the percentage of the group that watched none of the three sports during the last year.

Exercise 2. *An insurance company examines its pool of auto insurance customers and gathers the following information:*

(i) *All customers insure at least one car.*

(ii) *70% of the customers insure more than one car.*

(iii) *20% of the customers insure a sports car.*

(iv) *Of those customers who insure more than one car, 15% insure a sports car.*

Find the percentage of customers who insure exactly one car and that car is not a sports car.

Exercise 3. *For a bill to come before the president of the United States, it must be passed by both the House of Representatives and the Senate. Assume that, of the bills presented to these two bodies,* 60 *percent pass the House,* 80 *percent pass the Senate, and* 90 *percent pass at least one of the two. Calculate the probability that the next bill presented to the two groups will come before the president.*

Exercise 4. *An insurance company estimates that 40% of policyholders who have only an auto policy will renew next year and 60% of policyholders who have only a homeowners policy will renew next year. The company estimates that 80% of policyholders who have both an auto and a homeowners policy will renew at least one of those policies next year.*

Company records show that 65% of policyholders have an auto policy, 50% of policyholders have a homeowners policy, and 15% of policyholders have both an auto and a homeowners policy.

Using the company's estimates, calculate the percentage of policyholders that will renew at least one policy next year.

Exercise 5. *In a fierce battle, not less than* 70 *percent of the soldiers lost one eye, not less than* 75 *percent lost one ear, not less than* 80 *percent lost one hand, and not less than* 85 *percent lost one leg. What is the minimal possible percentage of those who simultaneously lost one ear, one eye, one hand, and one leg?*

Exercise 6. *A survey of 1000 people determines that 80% like walking and 60% like biking, and all like at least one of the two activities. How many people in the survey like biking but not walking?*

Exercise 7. *A pair of dice is rolled. What is the probability that the second die lands on a higher value than does the first?*

Exercise 8. *A three-person basketball team consists of a guard, a forward, and a centre.*

(a) If a person is chosen at random from each of three different such teams, what is the probability of selecting a complete team?

(b) What is the probability that all 3 players selected play the same position?

Exercise 9. *Let A and B be events such that $P(A) = .7$ and $P(B) = .9$. Calculate the largest possible value of $P(A \cup B) - P(A \cap B)$.*

Exercise 10. *Event C is a subevent of $A \cup B$. Which of the following must be true.*

I) $A \cup C = B \cup C$

II) $P(C) = P(A \cap C) + P(B \cap C) - P(A \cap B \cap C)$

III) $A^c \cap C \subset B$

Exercise 11. *Let A and $A_1, A_2, \ldots$ be any events in Ω. Show the following identities called* **distributive laws**

(i) $A \cap (\cup_{i=1}^{\infty} A_i) = \cup_{i=1}^{\infty} (A \cap A_i)$

(ii) $A \cup (\cap_{i=1}^{\infty} A_i) = \cap_{i=1}^{\infty} (A \cup A_i)$

Exercise 12. *Let A, B, and C be arbitrary events in Ω. Which of the following is correct or incorrect*

(i) $(A \setminus B) \cup B = (A \cap B^c) \cup B$

(ii) $(A \cup B) \setminus A = (A \cup B) \cap A^c$

(iii) $(A \cap B) \cap (A \setminus B) = (A \cap B) \cap (A \cap B^c)$

(iv) $(A \cup B) \cap (B \cup C) \cap (C \cup A) = (A \cap B) \cup (B \cap C) \cup (C \cap A)$

Exercise 13. *A deck of ordinary cards is shuffled and* 13 *cards are dealt. What is the probability that the last card dealt is an ace?*

Exercise 14. *In arranging people around a circular table, we take into account their seats relative to each other, not the actual position of any one person. Show that n people can be arranged around a circular table in $(n-1)!$ ways.*

Exercise 15. *Consider the problem of finding the probability of more than one coincidence of birthdays in a group of n people. These include, for example, three people with the same birthday, or two pairs of people with the same birthday, or larger coincidences. Show how you could compute this probability. Use a calculator or a computer to find the smallest number of people for which it would be a favorable bet that there would be more than one coincidence of birthdays.*

Exercise 16. *Charles claims that he can distinguish between beer and ale 75 percent of the time. Ruth bets that he cannot and, in fact, just guesses. To settle this, a bet is made: Charles is to be given ten small glasses, each having been filled with beer or ale, chosen by tossing a fair coin. He wins the bet if he gets seven or more correct. Find the probability that Charles wins if he has the ability that he claims. Find the probability that Ruth wins if Charles is guessing.*

Exercise 17. *There are n socks, three of which are red, in a drawer. What is the value of n if, when 2 of the socks are chosen randomly, the probability that they are both red is $1/2$?*

Exercise 18. *There are 12 strangers in a room. What is the probability that no two of them celebrate their birthday in the same month?*

Exercise 19. *A poker hand is a set of 5 cards randomly chosen from a deck of 52 cards. Find the probability of a*
(a) royal flush (ten, jack, queen, king, ace in a single suit).
(b) straight flush (five in a sequence in a single suit, but not a royal flush).
(c) four of a kind (four cards of the same face value).
(d) full house (one pair and one triple, each of the same face value).
(e) flush (five cards in a single suit but not a straight or royal flush).
(f) straight (five cards in a sequence, not all the same suit). (Note that in straights, an ace counts high or low.)

Exercise 20. *Let $A_1, A_2, \dots, A_n$ be events in the sample space Ω with probability measure P. Show that*
a) $P(A_1 \cup A_2 \cup \cdots \cup A_n) \leq P(A_1) + P(A_2) + \cdots + P(A_n)$;
b) $P(A_1 \cap A_2 \cap \cdots \cap A_n) \geq P(A_1) + P(A_2) + \cdots + P(A_n) - (n-1)$;
c) If $P(A_i) = 1$ *for all* $i = 1, 2, \dots, n$ *then* $P(A_1 \cap A_2 \cap \cdots \cap A_n) = 1$.

Exercise 21. *a) Show that*

$$\sum_{k=0}^{n} \binom{n}{k} = 2^n.$$

b) Show that

$$\sum_{k=0}^{n} (-1)^k \binom{n}{k} = 0.$$

Exercise 22. *a) By considering the selection of k students from a class with m boys and n girls, show that*

$$\binom{n+m}{k} = \binom{n}{0}\binom{m}{k} + \binom{n}{1}\binom{m}{k-1} + \cdots + \binom{n}{k}\binom{m}{0}$$

where $\binom{i}{j} = 0$ *if* $j > i$.
b) Show that

$$\binom{2n}{n} = \binom{n}{0}^2 + \binom{n}{1}^2 + \cdots + \binom{n}{n}^2.$$

Exercise 23. *a) Is the set of all finite words written with the letters H and T finite, countable, or uncountable? For example: T, HT, TTH are three finite words.*
b) Is the set of all infinite sequences of the letters H and T finite, countable, or uncountable? One infinite sequence could be HTHHTTT...

Exercise 24. *An urn contains* n *red and* m *blue balls.*
a) They are withdrawn one at a time. What is the probability that there are r *red balls in the first* k *withdrawals? (*$r \leq n$ *and* $k \leq m + n$*)*
b) They are withdrawn one at a time until a total of r *red balls have been withdrawn (*$r \leq n$*). Find the probability that a total of* k *balls are withdrawn.*

Exercise 25. *If* 10 *married couples are seated at random at a round table, compute the probability that no wife sits next to her husband.*