STAT 2857 — Assignment 2

Exercise 1. *How many numbers can be made each using all the digits 1, 2, 3, 4, 4, 5, 5, 5?*

Exercise 2. *Five persons, A, B, C, D, and E, are going to speak at a meeting.*
a) In how many orders can they take their turns if B must speak after A?
b) How many if B must speak immediately after A?

Exercise 3. *In how many ways can the letters of the word*

MUHAMMADAN

be arranged without letting three letters that are alike come together?

Exercise 4. *If 10 married couples are seated at random at a round table, compute the probability that each wife sits next to her husband.*

Exercise 5. *A class of* 30 *children comes into a store that sells ice cream cones. There are five flavours available. Each child gets just one cone. The management of the store does not care who gets which flavour. From the point of view of the management, how many different selections of flavours are possible?*

Exercise 6. *In how many ways can five apples and six oranges be distributed among seven children, disregarding the order of distribution?*

Exercise 7. *How many ways are there to choose three letters from the phrase*

MISS MISSISSIPPI NEVER EVER SIMPERS

ignoring the order of selection.

Exercise 8. *Let* A, B *and* C *be events such that* $\mathbb{P}\{A|C\} = 0.05$ *and* $\mathbb{P}\{B|C\} = 0.05$. *One of the following statements must be true, which one?*
A) $\mathbb{P}\{A \cap B|C\} = (0.05)^2$ *B)* $\mathbb{P}\{A^c \cap B^c|C\} \geq 0.90$
C) $\mathbb{P}\{A \cup B|C\} \leq 0.05$ *D)* $\mathbb{P}\{A \cup B|C^c\} \geq 1 - (0.05)^2$
E) $\mathbb{P}\{A \cup B|C\} \geq 0.10$

Exercise 9. *Either Alice or Betty is equally likely to be in the shower. Then you hear the showerer singing. You know that Alice always sings in the shower, while Betty only sings 1/4 of the time. What is the probability that Alice is in the shower?*

Exercise 10. *A friend has three cards: one red on both sides, one black on both sides, and one red on one side and black on the other. She mixes them up in a bag, draws one at random, and places it on the table with a red side showing. What is the probability that the other side is also red?*

Exercise 11. *You roll a single six-sided die, and then flip a coin the number of times showing on the die. The coin comes up heads every time. What are the probabilities that the die showed 1, 2, 3, 4, 5, and 6, respectively?*

Exercise 12. *Students* A, B *and* C *each independently answer a question on a test. The probability of getting the correct answer is 0.9 for* A*, 0.7 for* B *and 0.4 for* C*. If two of them get the correct answer, what is the probability* C *was the one with the wrong answer?*

Exercise 13. *From a pool of* N *questions* n *are easy. Two people draw a question without returning it. Who has a higher chance of drawing an easy question?*

Exercise 14. *A test for a disease correctly diagnoses a diseased person as having the disease with probability .85. The test incorrectly diagnoses someone without the disease as having the disease with a probability of .10. If 1% of the people in a population have the disease, what is the chance that a person from this population who tests positive for the disease actually has the disease?*

Exercise 15. *Events* A *and* B *are independent with* $\mathbb{P}\{A\} = 0.3$ *and* $\mathbb{P}\{B\} = 0.2$*. Find* $\mathbb{P}\{A \cup B\}$.

Exercise 16. *Let* E *and* F *be independent with* $E = A \cup B$*, and* $F = A \cap B$*. Prove that either* $\mathbb{P}\{A \cap B\} = 0$ *or else* $\mathbb{P}\{A^c \cap B^c\} = 0$.

Exercise 17. *Let the events* A*,* B*, and* C *be independent. Are the events* $A \cup B$ *and* C *independent? Why?*

Exercise 18. *In a certain factory 58% of the workers are male, 65% are married, and 25% are single females.*

(i) What percentage of the workers are married females?

(ii) What percentage of the workers are single males?

(iii) Is martial status independent of gender?

Exercise 19. *An insurance company insures E drivers under age 24 and D drivers over 24 years old. Of these drivers, e under 24 and d over 24 had an accident in a 1 year period. A driver insured by this company is chosen at random. Let A be the event that this driver is under 24 and B be the event that this driver did not have an accident in a 1 year period. Find the necessary and sufficient condition on e, d, E, and D so that A and B are independent events.*

Exercise 20. *Let A, B and C be events such that A and B are independent, B and C are mutually exclusive, $\mathbb{P}\{A\} = \frac{1}{4}$, $\mathbb{P}\{B\} = \frac{1}{6}$, and $\mathbb{P}\{C\} = \frac{1}{2}$. Find $\mathbb{P}\{(A \cap B)^c \cup C\}$.*

Exercise 21. *Flip a fair coin repeatedly. What is the probability that the first sequence of heads is exactly two heads long?*

Exercise 22. *a) Show that for any events A, B, and C we have*

$$P(A|B) = P(A|B \cap C)P(C|B) + P(A|B \cap C^c)P(C^c|B).$$

b) Prove or give a counterexample. If A and B are independent, then they are conditionally independent given C.

Exercise 23. *There are n balls in a box. Some of them are white. Assume that a priori it is equally likely that there is any number of white balls in the box (between 0 and n). A ball drawn from the box turns out to be white. How many white balls are there in the box most probably?*