STAT 2857 — Assignment 5

Problem 1. Let X have a geometric distribution with $f(x) = p(1-p)^x$, $x = 0, 1, 2, \ldots$ Find the probability function of R, the reminder when X is divided by 4.

Problem 2. The joint probability mass function of (X, Y) is given by

$f(x,y)$		X = 0	1	2
Y	0	.15	.1	.05
	1	.35	.2	.15

(i) Are X and Y independent? Why?

(ii) Find $\mathbb{P}\{X > Y\}$ and $\mathbb{P}\{X = 1 | Y = 0\}$.

Problem 3. Let X_1, X_2, X_3 be a independent and identically distributed random variables with a common discrete probability function

$$p(x) = \begin{cases} \frac{1}{3}, & x = 0 \\ \frac{2}{3}, & x = 1 \\ 0 & \text{otherwise.} \end{cases}$$

Determine the moment generating function, $M(t)$, of $Y = X_1 X_2 X_3$.

Problem 4. Let X and Y be random variables with joint probability mass function $f(x,y) = k \cdot \frac{2^{x+y}}{x!y!}$ for $x = 0, 1, 2, \ldots$ and $y = 0, 1, 2, \ldots$, where k is a positive constant.

(i) Derive the marginal probability mass function of X.

(ii) Evaluate k.

(iii) Are X and Y independent? Explain.

(iv) Derive the probability mass function of $T = X + Y$.

Problem 5. Calculate the moment generating function of
a) a normal random variable with parameters μ and σ^2;
b) a Gamma random variable with parameters $k > 0$ and $\lambda > 0$.

Problem 6. The internal auditing staff of a local manufacturing company performs a sample audit each quarter to estimate the proportion of accounts that are delinquent more than 90 days overdue. The historical records of the company show that over the past 8 years, 14 percent of the accounts are delinquent. For this quarter, the auditing staff randomly selected 250 customer accounts. What is the probability that at least 30 accounts will be classified as delinquent?

Problem 7. An insurance company insures a large number of drivers. Let X be the random variable representing the company's losses under collision insurance, and let Y represent the company's losses under liability insurance. X and Y have joint density function

$$f(x,y) = \begin{cases} \frac{2x+2-y}{4}, & 0 < x < 1,\, 0 < y < 2 \\ 0 & \text{otherwise.} \end{cases}$$

What is the probability that the total loss is at least 1?

Problem 8. Generally 5% of all students of STAT 2857 will fail the final exam. Suppose that the total number of students taking STAT 2857 is 550. Let X represent the number (out of 550 students) who will pass the final exam. What are the values of the mean and the standard deviation of X?

Problem 9. In a model for hospital room charges X and hospital surgical charges Y for a particular type of hospital admission, the region of probability (after scaling units) is $0 \leq y \leq 2x+1 \leq 3$. The joint density function of X and Y is $f(x,y) = .3(x+y)$. Find the expected excess of surgical charges over room charges for an admission.

Problem 10. A man and a woman agree to meet at a certain location about 12:30 P.M. If the man arrives at a time uniformly distributed between 12:15 and 12:45 and if the woman independently arrives at a time uniformly distributed between 12:00 and 1 P.M., find the probability that the first to arrive waits no longer than 5 minutes. What is the probability that the man arrives first?

Problem 11. The joint probability density function of X and Y is given by

$$f(x,y) = \frac{6}{7}\Big(x^2 + \frac{xy}{2}\Big) \quad \text{for} \quad 0 < x < 1, 0 < y < 2$$

and 0 otherwise.
a) Verify that this is indeed a joint density function.
b) Compute the density function of X.
c) Find $P[X > Y]$.
d) Find $P[Y > 1/2 | X < 1/2]$.
e) Find $E[X]$.
f) Find $E[Y]$.

Problem 12. Suppose that X, Y, Z, are independent random variables, each being uniformly distributed over $[0,1]$.
a) What is the joint cumulative distribution function of X, Y, Z?
b) What is the probability that all of the roots of the equation $Xa^2 + Ya + Z = 0$ are real?

Problem 13. Suppose that n points are independently chosen at random on the perimeter of a circle, and we want the probability that they all lie in some semicircle. (That is, we want the

probability that there is a line passing through the center of the circle such that all the points are on one side of that line.) Let P_1,..., P_n denote the n points. Let A denote the event that all the points are contained in some semicircle, and let A_i be the event that all the points lie in the semicircle beginning at the point P_i and going clockwise for 180°, $i = 1$,...,n.
a) Express A in terms of the A_i.
b) Are the A_i mutually exclusive?
c) Find $P(A)$.

Problem 14. The weight of males in Canada is normally distributed with mean 73.5 kgs and standard deviation 11.3 kgs. The weight of females in Canada is normally distributed with mean 57.2 kgs and standard deviation of 8.8 kgs. What is the probability that a couple (male+female) the combined weight is bigger than 150 kgs.?

Problem 15. Let X and Y be independent random variables with X being uniformly distributed in $[0, 1]$ and Y being exponential with parameter λ. Find the probability density function of
a) $X + Y$;
b) X/Y.

Problem 16. Show that if X is a positive random variable with $E[X] = 0$, then $P(X = 0) = 1$.

Problem 17. Let X be a random variable such that $P(X = 0) < 1$. Is it possible for a moment generating function of X to be a polynomial $a_n t^n + a_{n-1} t^{n-1} + \cdots + a_1 t + a_0$? Why?

Problem 18. You have 2 buckets. One full of white marbles and the other full of black marbles (equal amounts). How do you allocate the marbles into two buckets in a way that maximizes your probability of picking 2 white ones when you pick 1 marble from each bucket?