STAT 2857 — Formula Sheet for the Final Exam

- For any numbers $a_1, a_2, \ldots, a_r$ we have

$$(a_1 + a_2 + \cdots + a_r)^n = \sum_{\substack{(n_1,n_2,\ldots,n_r) \text{ s.t.} \\ n_1+n_2+\cdots+n_r=n \\ n_1\geq 0,\ldots,n_r\geq 0}} \frac{n!}{n_1!n_2!\cdots n_r!} a_1^{n_1} a_2^{n_2} \cdots a_r^{n_r}.$$

- Discrete uniform r.v. $X \in \{1, 2, \ldots, n\}$, $P(X = k) = 1/n$, $\mathbf{E}[X] = (n + 1)/2$, $\mathrm{Var}[X] = (n^2 - 1)/12$, $M(t) = (e^{t(n+1)} - e^t)/n(e^t - 1)$.

- Continuous uniform r.v. $X \in [a, b]$,

$$f(x) = \begin{cases} 1/(b-a) & \text{if } x \in [a,b], \\ 0 & \text{otherwise,} \end{cases} \qquad F(x) = \begin{cases} 0 & \text{if } x < a, \\ (x-a)/(b-a) & \text{if } a \leq x < b, \\ 1 & \text{if } b \leq x, \end{cases}$$

$\mathbf{E}[X] = (a + b)/2$, $\mathrm{Var}[X] = (a - b)^2/12$, $M(t) = (e^{tb} - e^{ta})/t(b - a)$.

- Binomial r.v. $X \in \{0, 1, \ldots, n\}$, $P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$, $E[X] = np$, $\mathrm{Var}[X] = np(1 - p)$, $M(t) = (pe^t + 1 - p)^n$

- Poisson r.v. $X \in \{0, 1, 2, \ldots\}$, $P(X = k) = e^{-\lambda} \frac{\lambda^k}{k!}$, $E[X] = \lambda$, $\mathrm{Var}[X] = \lambda$, $M(t) = e^{\lambda(e^t - 1)}$

- Exponential r.v. $X \in [0, \infty)$, $E[X] = 1/\lambda$, $\mathrm{Var}[X] = 1/\lambda^2$,

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}, \quad M(t) = \begin{cases} -\frac{\lambda}{t-\lambda} & \text{if } t < \lambda \\ \infty & \text{if } t \geq \lambda. \end{cases}$$

- Geometric r.v. $X \in \{1, 2, \ldots\}$, $P(X = k) = (1 - p)^{k-1} p$, $E[X] = 1/p$, $\mathrm{Var}[X] = (1 - p)/p^2$,

$$M(t) = \begin{cases} \frac{pe^t}{1-e^t(1-p)} & \text{if } e^t(1-p) < 1 \\ \infty & \text{if } e^t(1-p) \geq 1. \end{cases}$$

- Negative binomial r.v. $X \in \{r, r + 1, r + 2, r + 3, \ldots\}$, $P(X = k) = \binom{k-1}{r-1} (1 - p)^{k-r} p^r$, $E[X] = r/p$, $\mathrm{Var}[X] = r(1 - p)/p^2$,

$$M(t) = \begin{cases} \left(\frac{pe^t}{1-e^t(1-p)}\right)^r & \text{if } e^t(1-p) < 1 \\ \infty & \text{if } e^t(1-p) \geq 1, \end{cases}$$

- Hypergeometric r.v. $X \in \{\max\{0, n + m - N\}, \ldots, \min\{n, m\}\}$, $P(X = k) = \frac{\binom{m}{k}\binom{N-m}{n-k}}{\binom{N}{n}}$, $E[X] = \frac{nm}{N}$, $\mathrm{Var}[X] = \frac{N-n}{N-1} \frac{nm}{N} \frac{N-m}{N}$.

- Normal r.v. $X \in \mathbb{R}$, $f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$, $E[X] = \mu$, $\mathrm{Var}[X] = \sigma^2$, $M(t) = e^{\left(\mu t + \frac{\sigma^2 t^2}{2}\right)}$.

- The gamma function is defined by $\Gamma(t) = \int_0^\infty e^{-x} x^{t-1}\, dx$.

- Gamma r.v. $X \in (0, \infty)$, $E[X] = k/\lambda$, $\mathrm{Var}[X] = k/\lambda^2$,

$$f(x) = \begin{cases} \frac{\lambda e^{-\lambda x}(\lambda x)^{k-1}}{\Gamma(k)} & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases}, \quad M(t) = \frac{1}{(1 - t/\lambda)^k} \text{ for } t < \lambda.$$

- The c.d.f. and the p.d.f. of $X + Y$ are

$$F_{X+Y}(t) = \int_{-\infty}^{\infty} f_Y(y) F_X(t - y)\, dy, \quad f_{X+Y}(t) = \int_{-\infty}^{\infty} f_Y(y) f_X(t - y)\, dy.$$

Table A.4 (Normal table)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998