Assignment 2: The GI/M/1 Queue, Queueing Networks, Priority Queues
Due: February 19

1. Using the method of generating function show that for GI/M/1 system
$B(z) = A^*(\mu - \mu z)$

2. Consider GI/M/1 queue where arrivals are Erlang with parameter (2, λ), hence $A^*(s) = (\frac{\lambda}{\lambda + s})^2$.
   a) Find the solution of the equation $a = A^*(\mu - \mu a)$
   b) Find π_0. What does π_0 represent?
   c) Find the queue length distribution $\pi_j, j = 1, 2, \ldots$.

3. Consider GI/M/1 queue. Assume that the customer interarrival time is a discrete random variable with possible values 1, 2,…, 6 and respective probabilities 0.1, 0.2, 0.3, 0.2 ,0.1, 0.1. Hence the mean interarrival time is 3.3. Assume that the mean service time is 3.0.

Use the program hw4_28a.exe, entering the mean service time and the tolerance of the numerical solution ε=0.00001.

(a) Solve *B(z)* = *z* for $0 \le z \le 1$. Find the queue length distribution $\{\pi_j\}$ before an arrival.

(b) Plot $\{\pi_j\}$.

(c) Assume that the mean service time is 4.0. Verify that the solution to *B(z)* = *z* for $0 \le z \le 1$ is 1.

(d) Assume that the mean service time is 2.0. Find the distribution $\{\pi_j\}$.

(f) Compare the results (a), (c) and (d).

(g) Consider example 3d again. Assume now that the tolerance ε = 0.001 instead ε = 0.00001. Explain the effect of the tolerance.

4. Suppose we have a computer system that consists of a CPU and two I/O devices and that maximal multiprogramming level is 4. Thus *M* = 3, *N*= 4. Suppose, also, that the average CPU time needed per job is 28 msec while the average 40 msec and 420 msec of I/O devices are required. Thus, μ_1 = 1/28 msec^{-1} , μ_2 = 1/40 msec^{-1} and μ_3 = 1/420 msec^{-1}.
Suppose that all service times are exponential. The probability that a program will request service from the *i*th device after completing a CPU processing interval is p_i , where $p_1 + p_2 + p_3 = 1$.
Let $p_1 = 0.1$, $p_2 = 0.7$, $p_3 = 0.2$.

(a) Find the CPU utilization for 1, 2, 3, and 4 levels of multiprogramming.

(b) Find the throughput at each server and the overall response time of the system for 4 level of multiprogramming.

© Find[1] the performance measures and test your work.

5. The Helpful Hardware Store is a small place, with just one person serving. There are two types of customers. Purchasers actually buy something, with a service time exponentially distributed and mean 150 sec. Enquirers have a simple query, with a service time exponentially distributed and mean 12 sec. On average one purchaser and two enquirers arrive every 5 min. The store owner believes he can improve service by having a separate queue for enquirers and by giving enquirers priority. Calculate[1] the mean waiting times for both categories of customers with FCFS, preemptive and non-preemptive priority.

6. Consider the system M/G/1. There are four classes of customers. Class 1 has the highest priority, class 2 has the next highest priority, and so on. The arrival rate λ = 0.04 sec^{-1} , mean service time 1/μ = 4.0 sec and coefficient of variation for service time c_s = 1.5 are the same for all customers. Compare[1] the mean waiting times for preemptive and non-preemptive priority scheduling.

[1] Use the program ZEDNET.EXE.