CHAPTER 10

A note on computing in time series situations

The **Minitab** software is used here for calculating the autocorrelation function of time series observations and for fitting the autoregressive integrated moving average (ARIMA) models in Chapter 10. The class of ARIMA models includes the autoregressive, random walk, and noisy random walk models discussed in Chapter 10. The Minitab ARIMA routine also facilitates the computation of the predictions and prediction intervals.

Combined regression time series models can be estimated within the **SCA** software or within the econometrics software **EVIEWS**. Contact information for these two software providers are:

- SCA: Scientific Computing Associates Corp.,1410 N. Harlem Avenue, River Forest, IL 60305. www.scausa.com.
- EVIEWS: QMS (Quantitative Micro Software), 4521 Campus Drive, Irvine, CA, 92612. www.eviews.com

For SCA one needs to construct a text file macro which is then executed by the software. The output can be saved into a file. Here we list the text file macro for Exercise 10.13.

```
==MACRO
Input variables are year quarter FTEShare Car FTEComm.
      1952  3     112.7 105761     96.21
      1952  4     115.0 121874     93.74
      1953  1     121.4 126260     91.37
      …
      …
      1967  2     343.1 393808     79.90
      1967  3     360.8 375968     78.70
      1967  4     397.8 381692     81.50
end
print variables are year quarter FTEShare Car FTEComm.
Utsmodel name is m1. @
Model is FTEShare((1-B)) = (w1*B**6)Car((1-B)) @
+ (w2*B**7)FTEComm((1-B)) + (1-theta*B)noise.
```

Model m1 considers the differences of the response and the regressor variables. The regression model relates the differences of the response to the differences of Car (with lag 6) and the differences of FTECom (with lag 7). A first order moving average model is taken as the error model.

```
Uestim m1. Method is EXACT. Hold residuals(resid1).
```

```
Acf variable is resid1.
Utsmodel name is m2. @
Model is FTEShare((1-B)) = (w1*B**6)Car((1-B)) @
+ (w2*B**7)FTEComm((1-B)) + 1/(1-phi*B)noise.
```

Model m2 considers differences of the response and the regressor variables. A first order autoregressive model is used as the error model.

```
Uestim m2. Method is EXACT. Hold residuals(resid2).
Acf variable is resid2.
RETURN
```

Many options are available within SCA. See the SCA on-line help for further discussion and examples.

A short primer on the backshift operator

The backshift operator B simplifies the notation of time series models. When applied to a time series y_t, the backshift operator shifts the time index by one unit. That is,

$$By_t = y_{t-1}, B^2 y_t = y_{t-2}, B^3 y_t = y_{t-3}, \text{ and so on.}$$

Similarly,

$$Bx_t = x_{t-1}, B^2 x_t = x_{t-2}, B^3 x_t = x_{t-3}, \text{ and so on.}$$

First differences of a time series can be written as $y_t - y_{t-1} = y_t - By_t = (1-B)y_t$.

Second differences (the difference of differences) as

$$y_t - y_{t-1} - (y_{t-1} - y_{t-2}) = (1-B)y_t - (1-B)y_{t-1} = (1-B)(y_t - y_{t-1}) = (1-B)^2 y_t$$

The first order moving average model can be written as

$$\varepsilon_t = a_t - \theta a_{t-1} \quad \text{or} \quad \varepsilon_t = (1-\theta B)a_t.$$

The first order autoregressive model can be written as

$$\varepsilon_t = \phi\varepsilon_{t-1} + a_t \quad \text{or} \quad \varepsilon_t - \phi B\varepsilon_t = a_t \quad \text{or} \quad (1-\phi B)\varepsilon_t = a_t.$$

We can also write it as

$$\varepsilon_t = \frac{1}{1-\phi B}a_t = (1+\phi B + \phi^2 B^2 + \ldots)a_t = a_t + \phi a_{t-1} + \phi^2 a_{t-2} + \ldots \quad .$$

The noisy random walk also known as the ARIMA(0,1,1) model,

$\varepsilon_t - \varepsilon_{t-1} = a_t - \theta a_{t-1}$, can be written as $(1-B)\varepsilon_t = (1-\theta B)a_t$. Or, as $\varepsilon_t = \frac{1-\theta B}{1-B}a_t$.

Regression models with (first-order) moving average errors

$$y_t = \beta_0 + \beta_1 x_t + \varepsilon_t \quad \text{with} \quad \varepsilon_t = (1-\theta B)a_t$$

can be combined as

$$y_t = \beta_0 + \beta_1 x_t + (1-\theta B)a_t.$$

Regression models with (first-order) autoregressive errors

$$y_t = \beta_0 + \beta_1 x_t + \varepsilon_t \quad \text{with} \quad (1-\phi B)\varepsilon_t = a_t$$

can be combined as

$$y_t = \beta_0 + \beta_1 x_t + \frac{1}{1-\phi B}a_t.$$

Regression models with noisy random walk errors

$$y_t = \beta_0 + \beta_1 x_t + \varepsilon_t \quad \text{with} \quad (1-B)\varepsilon_t = (1-\theta B)a_t$$

can be combined as

$$y_t = \beta_0 + \beta_1 x_t + \frac{1-\theta B}{1-B} a_t \, .$$

Alternatively, this model can be written as a regression of differences,

$$(1-B)y_t = \beta_1(1-B)x_t + (1-\theta B)a_t \, ;$$

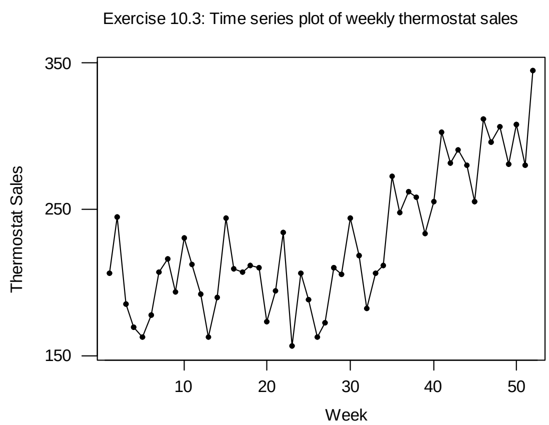
the constant disappears as $(1-B)\beta_0 = \beta_0 - \beta_0 = 0$.

10.1 $\ln(1+r_t) \approx \ln(1) + (r_t - 0)\frac{\partial \ln(1+r_t)}{\partial r_t}\Big|_{r_t=0} = r_t \frac{1}{1+r_t}\Big|_{r_t=0} = r_t$

$\ln(y_t) = \ln[y_{t-1}(1+r_t)] = \ln(y_{t-1}) + \ln(1+r_t) \approx \ln(y_{t-1}) + r_t$

10.2 Write out the matrices L' and L, form the matrix product $L'L$, and show that it equals $(1-\phi^2)V^{-1}$.

10.3 (a) The time series plot of the data is given below.



(b) The MINITAB output of the regression of sales on time, $y_t = \beta_0 + \beta_1 t + \varepsilon_t$, is shown below. The predictions and the 95 percent prediction intervals for the next three observations are calculated from the results in Section 4.3.2.

```
The regression equation is
Sales = 166 + 2.32 Time

Predictor        Coef     SE Coef          T        P
Constant      166.396       8.760      19.00    0.000
Time           2.3247      0.2876       8.08    0.000

S = 31.13       R-Sq = 56.6%     R-Sq(adj) = 55.8%

Analysis of Variance

Source            DF          SS          MS         F        P
Regression         1       63299       63299     65.32    0.000
Residual Error    50       48451         969
Total             51      111750
```

Prediction for the next period (time = 53):
Prediction: $y_{52}(1) = 166.396 + (2.325)(53) = 289.60$
Prediction interval: 224.65, 354.56
Predictions and prediction intervals can be obtained with the Minitab option in the "regress" command. Alternatively, one can calculate them from the results in Chapter 2,

$$y_{52}(1) \pm 2.0086\sqrt{969}\sqrt{1 + \frac{1}{52} + \frac{(53-26.5)^2}{11{,}713}} ,$$

where 2.0086 is the 97.5th percentile of the t-distribution with 50 degrees of freedom, $26.5 = (1/52)\sum_{t=1}^{52} t$ and $11{,}713 = \sum_{t=1}^{52}(t-26.5)^2$.

Prediction for two periods ahead (time = 54):
Prediction: $y_{52}(2) = 166.396 + (2.325)(54) = 291.93$
Prediction interval: 226.84, 357.02

$$y_{52}(2) \pm 2.0086\sqrt{969}\sqrt{1 + \frac{1}{52} + \frac{(54-26.5)^2}{11{,}713}}$$

Prediction for three periods ahead (time = 55):
Prediction: $y_{52}(3) = 166.396 + (2.325)(55) = 294.25$
Prediction interval: 229.02, 359.49

$$y_{52}(3) \pm 2.0086\sqrt{969}\sqrt{1 + \frac{1}{52} + \frac{(55-26.5)^2}{11{,}713}}$$

(c) The Durbin-Watson test statistic is 1.09, and far from the desired value 2. It is not acceptable. There is autocorrelation in the residuals. The first ten autocorrelations are given below (read across):

```
0.405962   0.257128   0.184543   0.192049    0.274287
0.401941   0.283462   0.172746   0.091004   -0.070815
```

The approximate standard error of an autocorrelation is given by $1/\sqrt{52} = 0.14$. Several of the autocorrelations exceed twice the standard error. The autocorrelations tend to be positive with a slow decay, indicating an autocorrelation problem and possible nonstationarity. A regression of sales on time, $y_t = \beta_0 + \beta_1 t + \varepsilon_t$, is definitely not an appropriate forecasting model. The plot of the residuals against time (given below) shows patterns.

(d) The mean of the first differences is 2.7255. This becomes the estimate of β_1 in the model $\Delta y_t = \beta_1 + a_t$. The standard deviation of the first differences is 32.51; this becomes the estimate of σ_a .

The forecasts for the next three observations are:

$y_{52}(1) = y_{52} + \hat{\beta}_1 = 345 + 2.73 = 347.73$

$y_{52}(2) = y_{52}(1) + \hat{\beta}_1 = 347.73 + 2.73 = 350.46$

$y_{52}(3) = y_{52}(2) + \hat{\beta}_1 = 350.46 + 2.73 = 353.19$

The prediction intervals are given by

$y_{52}(1) \pm (1.96)(32.51)$	or	347.73 ± 63.72
$y_{52}(2) \pm (1.96)(\sqrt{2})(32.51)$	or	350.46 ± 90.11
$y_{52}(3) \pm (1.96)(\sqrt{3})(32.51)$	or	353.19 ± 110.37

The first ten autocorrelations of the differenced series are given below (read across):

```
 -0.365082  -0.059187  -0.033625  -0.093252  -0.041308
  0.186040   0.048240  -0.038622   0.034502  -0.169835
```

The lag one autocorrelation exceeds twice its approximate standard error $1/\sqrt{51} = 0.14$. Hence this is not an appropriate forecasting model.

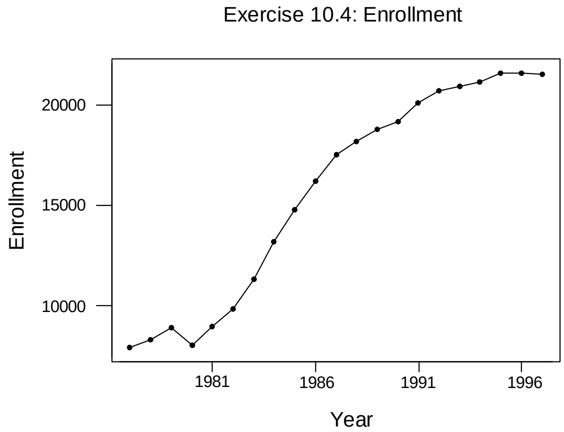
(e) The ARIMA time series procedure in MINITAB is used to estimate the noisy random walk model $\Delta y_t = y_t - y_{t-1} = \beta_1 + a_t - \theta a_{t-1}$. Using the MINITAB ARIMA command, we find

```
Estimates at each iteration
Iteration        SSE     Parameters
    0         49361.5    0.100    2.825
    1         45310.4    0.250    2.496
    2         42249.3    0.400    2.245
    3         39884.7    0.550    2.106
    4         38533.0    0.687    2.124
    5         38448.9    0.717    2.220
    6         38447.7    0.719    2.248
    7         38447.7    0.720    2.251
    8         38447.7    0.720    2.252
Relative change in each estimate less than  0.0010

Final Estimates of Parameters
Type          Coef     SE Coef         T        P
MA   1      0.7198      0.1010      7.13    0.000
Constant     2.252       1.127      2.00    0.051

Differencing: 1 regular difference
Number of observations:  Original series 52, after differencing 51
Residuals:    SS =  38356.2  (backforecasts excluded)
              MS =   782.8  DF = 49

Forecasts from period 52
                             95 Percent Limits
Period      Forecast        Lower         Upper
  53         313.544      258.696       368.392
  54         315.796      258.836       372.756
  55         318.048      259.052       377.045
```

The estimates are $\hat{\beta}_1 = 2.252$ and $\hat{\theta} = 0.72$. The forecasts and the 95 percent prediction intervals are part of the MINITAB output. The first ten autocorrelations of the residuals from this model are shown below. They are small (most of them smaller than their standard error), indicating that we have found an acceptable model.

```
0.066442  -0.067055  -0.127384  -0.104795   0.045999
0.283976   0.172438   0.061706  -0.010849  -0.161526
```

10.4 (a) The time series plot shows that the linear trend is not globally stable. The trend shifts over time. Hence a regression on time, $y_t = \beta_0 + \beta_1 t + \varepsilon_t$, is not appropriate. The residuals from the (incorrect) regression on time show (positive) autocorrelations and an unacceptable Durbin-Watson test statistic (0.26) that is considerably smaller than 2.




```
The regression equation is
enrollment = 6527 + 830 time

Predictor        Coef     SE Coef          T        P
Constant       6527.2       599.6      10.89    0.000
time           830.08       47.75      17.38    0.000

S = 1325        R-Sq = 94.1%     R-Sq(adj) = 93.8%

Analysis of Variance

Source            DF          SS          MS         F        P
Regression         1   530560905   530560905    302.14    0.000
Residual Error    19    33363694     1755984
Total             20   563924600

Durbin-Watson statistic = 0.26

First four autocorrelations of residuals

   0.779040   0.504676   0.191752  -0.088873
```

The predictions and 95% prediction intervals for the next three periods are given below. Because of the residual problems with this model, these predictions should not be used:

For the next period (time = 22): 24,789 and (21,745 to 27,833)
For two periods ahead (time = 23): 25,619 and (22,537 to 28,701)
For three periods ahead (time = 24): 26,449 and (23,327 to 29,571)

(b) The mean of the first differences is 682. This becomes the estimate of β_1. The standard deviation of the first differences is 654; this becomes the estimate of σ_ε .

The forecasts for the next three observations are:

$y_{21}(1) = y_{21} + \hat{\beta}_1 = 21{,}531 + 682 = 22{,}213$

$y_{21}(2) = y_{21}(1) + \hat{\beta}_1 = y_{21} + 2\hat{\beta}_1 = 22{,}213 + 682 = 22{,}895$

$y_{21}(3) = y_{21}(2) + \hat{\beta}_1 = y_{21} + 3\hat{\beta}_1 = 22{,}895 + 682 = 23{,}577$

The prediction intervals are given by

$y_{21}(1) \pm (1.96)(654)$ or $22{,}213 \pm 1{,}282$

$y_{21}(2) \pm (1.96)(\sqrt{2})(654)$ or $22{,}895 \pm 1{,}813$

$y_{21}(3) \pm (1.96)(\sqrt{3})(654)$ or $23{,}577 \pm 2{,}220$

The first four autocorrelations of the differenced series are given below (read across):

```
0.491156   0.393677   0.114746  -0.074641
```

The lag one autocorrelation exceeds twice its approximate standard error $1/\sqrt{20} = 0.22$.
This forecasting model is not appropriate.

(c) The regression of enrollment on the previous two enrollments (lag one and two), $y_t = \beta_0 + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \varepsilon_t$, is given below. The Durbin-Watson statistic is much better; it is close to the desired value 2. Also, the autocorrelations of the residuals are small. This model provides an appropriate forecasting method.

```
The regression equation is
enroll = 914 + 1.47 enroll-1 - 0.506 enroll-2

19 cases used 2 cases contain missing values

Predictor        Coef     SE Coef          T        P
Constant        914.4       477.6       1.91    0.074
enroll-1       1.4691      0.2147       6.84    0.000
enroll-2      -0.5061      0.2108      -2.40    0.029

S = 575.2       R-Sq = 98.8%     R-Sq(adj) = 98.6%

Analysis of Variance

Source            DF          SS          MS        F        P
Regression         2   431791259   215895629   652.54    0.000
Residual Error    16     5293676      330855
Total             18   437084935

Durbin-Watson statistic = 2.32

First four autocorrelations of the residuals:
```

```
  -0.168526   0.104467  -0.054733  -0.121096
```

The root mean square error from the second-order autoregression, $\sqrt{330{,}855} = 575$, is considerably smaller than the root mean square error of the regression on time in (a), $\sqrt{1{,}755{,}984} = 1{,}325$. The AR(2) model is preferable.

The forecasts can be obtained from:

$$y_{21}(1) = 914 + 1.47y_{21} - 0.51y_{20} = 914 + 1.47(21{,}531) - 0.51(21{,}624) = 21{,}536$$
$$y_{21}(2) = 914 + 1.47y_{21}(1) - 0.51y_{21} = 914 + 1.47(21{,}536) - 0.51(21{,}531) = 21{,}592$$
$$y_{21}(3) = 914 + 1.47y_{21}(2) - 0.51y_{21}(1) = 914 + 1.47(21{,}592) - 0.51(21{,}536) = 21{,}670$$

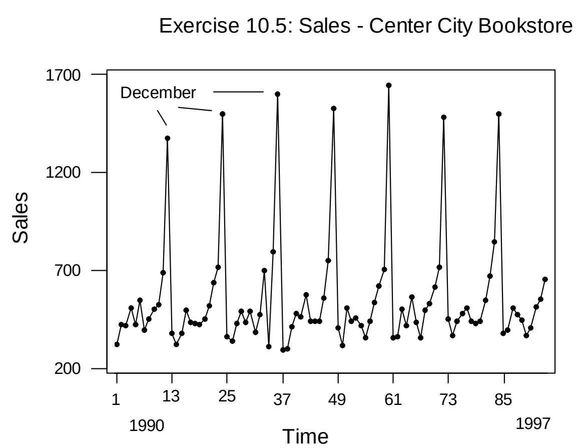
Another reasonable model for these data is the second difference model,

$$(y_t - y_{t-1}) - (y_{t-1} - y_{t-2}) = y_t - 2y_{t-1} + y_{t-2} = \varepsilon_t \quad .$$

It is a special case of the AR(2) model with $\phi_1 = 2$ and $\phi_2 = -1$. The forecasts are

$$y_{21}(1) = 2y_{21} - y_{20} = 2(21{,}531) - 21{,}624 = 21{,}438$$
$$y_{22}(2) = 2y_{21}(1) - y_{21} = 2(21{,}438) - 21{,}531 = 21{,}345$$
$$y_{21}(3) = 2y_{21}(2) - y_{21}(1) = 2(21{,}345) - 21{,}438 = 21{,}252$$

10.5 (a) A time series graph of the observations shows the high sales activity during December months. The question whether or not the data exhibit a trend component is difficult to answer from just the graph alone.



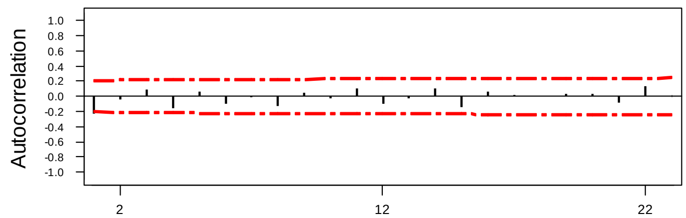
We consider a model with a linear time trend and monthly indicators that account for the seasonal pattern,

$$\text{Sales}_t = \beta_0 + \beta_1 t + \beta_2 \text{IndJan}_t + \beta_3 \text{IndFeb}_t + ... + \beta_{12} \text{IndNov}_t + \varepsilon_t \ .$$

The estimation results indicate a positive trend component. The probability value of the trend coefficient is 0.058, which indicates weak statistical significance. The magnitude of the trend coefficient, a 0.45 EURO increase per month, is of no practical importance. The coefficients of the indicators express differences in average sales for the various months and their base of comparison (December). For example, the value for January (-1,154) indicates that sales in January are on average 1,154 EUROs lower than those in December. The residuals from the regression are still autocorrelated, especially at lag 1; the lag one autocorrelation -0.23 exceeds twice its standard error, $1/\sqrt{94} = 0.10$. The Durbin-Watson statistic (2.45) is larger than 2, reflecting a negative lag one autocorrelation.

```
The regression equation is
Sales = 1500 + 0.449 Time - 1154 IndJan - 1169 IndFeb - 1073 IndMar
           - 1049 IndApr - 1057 IndMay - 1061 IndJun - 1126 IndJul
           - 1062 IndAug - 984 IndSep - 951 IndOct - 776 IndNov

Predictor        Coef     SE Coef          T        P
Constant      1500.18       25.68      58.42    0.000
Time           0.4487      0.2335       1.92    0.058
IndJan       -1154.47       31.66     -36.47    0.000
IndFeb       -1169.04       31.65     -36.94    0.000
IndMar       -1073.12       31.64     -33.91    0.000
IndApr       -1048.82       31.64     -33.15    0.000
IndMay       -1057.27       31.64     -33.42    0.000
IndJun       -1060.96       31.64     -33.54    0.000
IndJul       -1125.91       31.64     -35.59    0.000
IndAug       -1061.74       31.64     -33.56    0.000
IndSep        -983.94       31.64     -31.09    0.000
IndOct        -951.13       31.65     -30.05    0.000
IndNov        -776.41       32.67     -23.76    0.000

S = 61.13       R-Sq = 96.4%     R-Sq(adj) = 95.8%

Analysis of Variance

Source            DF          SS          MS         F        P
Regression        12     7992176      666015    178.25    0.000
Residual Error    81      302649        3736
Total             93     8294825

Durbin-Watson statistic = 2.45
```

Autocorrelation Function of Residuals



Lag	Corr	T	LBQ	Lag	Corr	T	LBQ	Lag	Corr	T	LBQ	Lag	Corr	T	LBQ
1	-0.23	-2.26	5.27	8	-0.13	-1.16	12.15	15	-0.14	-1.22	18.16	22	0.13	1.03	21.81
2	-0.05	-0.47	5.53	9	0.05	0.42	12.39	16	0.06	0.53	18.63	23	-0.00	-0.02	21.81
3	0.09	0.83	6.34	10	-0.03	-0.23	12.47	17	0.01	0.09	18.64				
4	-0.16	-1.45	8.85	11	0.10	0.88	13.59	18	-0.00	-0.04	18.65				
5	0.05	0.46	9.12	12	-0.10	-0.84	14.64	19	0.03	0.25	18.76				
6	-0.11	-0.96	10.31	13	-0.03	-0.25	14.74	20	0.03	0.26	18.88				
7	-0.02	-0.13	10.34	14	0.10	0.83	15.81	21	-0.09	-0.74	19.85				

(b) The autocorrelation function of the residuals from the model in (a) has a spike at lag one. This suggests a first-order moving average model for the errors. Alternatively, one could consider a first-order autoregressive model. We study both error models and show that the results for these two error models are very similar.

$$\text{MA(1): Sales}_t = \beta_0 + \beta_1 t + \beta_2 \text{IndJan}_t + \beta_3 \text{IndFeb}_t + ... + \beta_{12} \text{IndNov}_t + (1-\theta B) a_t$$

or,

$$\text{AR(1): Sales}_t = \beta_0 + \beta_1 t + \beta_2 \text{IndJan}_t + \beta_3 \text{IndFeb}_t + ... + \beta_{12} \text{IndNov}_t + \frac{1}{1-\phi B} a_t$$

We use SCA to estimate the models (alternatively, one could use Eviews). The results for MA(1) errors are shown first. The residuals from the revised model are uncorrelated. The lag one autocorrelation of the residuals is 0.10, and is well within one standard error. The trend coefficient is small and can be neglected for practical purposes. The seasonal component is very strong.

	PARAMETER LABEL	VARIABLE NAME	NUM./ DENOM.	FACTOR	ORDER	CONS- TRAINT	VALUE	STD ERROR	T VALUE
1	CNST		CNST	1	0	NONE	1500.8551	22.7676	65.92
2	B1	TIME	NUM.	1	0	NONE	.4363	.1545	2.82
3	B2	INDJAN	NUM.	1	0	NONE	-1154.6164	32.6848	-35.33
4	B3	INDFEB	NUM.	1	0	NONE	-1169.1776	29.3843	-39.79
5	B4	INDMAR	NUM.	1	0	NONE	-1073.2389	29.3796	-36.53
6	B5	INDAPR	NUM.	1	0	NONE	-1048.9253	29.3758	-35.71
7	B6	INDMAY	NUM.	1	0	NONE	-1057.3616	29.3727	-36.00
8	B7	INDJUN	NUM.	1	0	NONE	-1061.0479	29.3705	-36.13
9	B8	INDJUL	NUM.	1	0	NONE	-1125.9842	29.3691	-38.34
10	B9	INDAUG	NUM.	1	0	NONE	-1061.7955	29.3685	-36.15
11	B10	INDSEP	NUM.	1	0	NONE	-983.9818	29.3687	-33.50
12	B11	INDOCT	NUM.	1	0	NONE	-951.1681	29.3698	-32.39
13	B12	INDNOV	NUM.	1	0	NONE	-778.9498	33.9353	-22.95
14	THETA	SALES	MA	1	1	NONE	.2721	.0995	2.74

```
EFFECTIVE NUMBER OF OBSERVATIONS . .           94
R-SQUARE . . . . . . . . . . . . . . .       0.966
RESIDUAL STANDARD ERROR. . . . . . .  0.548881E+02

AUTOCORRELATIONS OF RESIDUALS

  1- 12     .01 -.04  .04 -.15 -.01 -.13 -.09 -.15  .01 -.00  .08 -.09
  ST.E.     .10  .10  .10  .10  .11  .11  .11  .11  .11  .11  .11  .11

 13- 24    -.04  .06 -.12  .04  .03  .01  .04  .03 -.05  .11  .02 -.02
  ST.E.     .11  .11  .11  .11  .11  .11  .11  .11  .11  .11  .12  .12
```

The results for AR(1) errors (shown below) are similar:

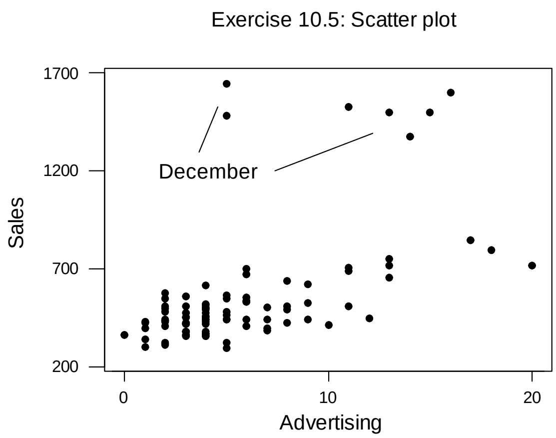
PARAMETER LABEL		VARIABLE NAME	NUM./ DENOM.	FACTOR	ORDER	CONS-TRAINT	VALUE	STD ERROR	T VALUE
1	CNST		CNST	1	0	NONE	1501.8372	23.1090	64.99
2	B1	TIME	NUM.	1	0	NONE	.4248	.1740	2.44
3	B2	INDJAN	NUM.	1	0	NONE	-1151.3433	33.8071	-34.06
4	B3	INDFEB	NUM.	1	0	NONE	-1170.5413	28.6817	-40.81
5	B4	INDMAR	NUM.	1	0	NONE	-1073.4953	29.6779	-36.17
6	B5	INDAPR	NUM.	1	0	NONE	-1049.4245	29.4279	-35.66
7	B6	INDMAY	NUM.	1	0	NONE	-1057.7898	29.4837	-35.88
8	B7	INDJUN	NUM.	1	0	NONE	-1061.4784	29.4667	-36.02
9	B8	INDJUL	NUM.	1	0	NONE	-1126.4000	29.4818	-38.21
10	B9	INDAUG	NUM.	1	0	NONE	-1062.2005	29.4317	-36.09
11	B10	INDSEP	NUM.	1	0	NONE	-984.3751	29.6497	-33.20
12	B11	INDOCT	NUM.	1	0	NONE	-951.5499	28.7184	-33.13
13	B12	INDNOV	NUM.	1	0	NONE	-779.1028	33.7655	-23.07
14	PHI	SALES	D-AR	1	1	NONE	-.2369	.1014	-2.34

```
EFFECTIVE NUMBER OF OBSERVATIONS . .           93
R-SQUARE . . . . . . . . . . . . . . .       0.965
RESIDUAL STANDARD ERROR. . . . . . .  0.553693E+02

AUTOCORRELATIONS OF RESIDUALS

  1- 12    -.02 -.09  .05 -.14  .01 -.10 -.08 -.14  .01  .01  .07 -.10
  ST.E.     .10  .10  .10  .10  .11  .11  .11  .11  .11  .11  .11  .11

 13- 24    -.04  .06 -.11  .04  .03  .01  .04  .02 -.05  .11  .02 -.03
  ST.E.     .11  .11  .11  .11  .11  .11  .11  .11  .11  .11  .12  .12
```

(c) A scatter plot of sales against advertising is shown below. Adding advertising expenditures to our earlier specification, we consider the model

$$\text{Sales}_t = \beta_0 + \beta_1 t + \beta_2 \text{IndJan}_t + \beta_3 \text{IndFeb}_t + ... + \beta_{12} \text{IndNov}_t + \beta_{13} \text{Adv}_t + (1-\theta B) a_t$$

The estimation results are given below. We find little evidence that advertising provides additional information. This finding can be explained by the fact that advertising is (partially) confounded with the seasonal pattern represented by the seasonal indicators.



The results show that

PARAMETER LABEL		VARIABLE NAME	NUM./ DENOM.	FACTOR	ORDER	CONS-TRAINT	VALUE	STD ERROR	T VALUE
1	CNST		CNST	1	0	NONE	1478.9299	33.7541	43.81
2	W1	TIME	NUM.	1	0	NONE	.4167	.1598	2.61
3	W2	INDJAN	NUM.	1	0	NONE	-1140.0198	36.2740	-31.43
4	W3	INDFEB	NUM.	1	0	NONE	-1149.4535	36.8407	-31.20
5	W4	INDMAR	NUM.	1	0	NONE	-1064.2115	30.9192	-34.42
6	W5	INDAPR	NUM.	1	0	NONE	-1034.2650	33.6153	-30.77
7	W6	INDMAY	NUM.	1	0	NONE	-1043.4471	33.1970	-31.43
8	W7	INDJUN	NUM.	1	0	NONE	-1045.3278	34.2302	-30.54
9	W8	INDJUL	NUM.	1	0	NONE	-1112.7960	32.8056	-33.92
10	W9	INDAUG	NUM.	1	0	NONE	-1046.2914	34.1008	-30.68
11	W10	INDSEP	NUM.	1	0	NONE	-971.7751	32.3066	-30.08
12	W11	INDOCT	NUM.	1	0	NONE	-942.0039	30.9651	-30.42
13	W12	INDNOV	NUM.	1	0	NONE	-785.5350	34.2710	-22.92
14	W13	ADV	NUM.	1	0	NONE	2.0413	2.3240	.88
15	THETA	SALES	MA	1	1	NONE	.2522	.0999	2.52

```
EFFECTIVE NUMBER OF OBSERVATIONS . .            94
R-SQUARE . . . . . . . . . . . . . .         0.966
RESIDUAL STANDARD ERROR. . . . . . .  0.546787E+02
```

AUTOCORRELATIONS OF RESIDUALS

1- 12	.00	-.03	.05	-.15	-.02	-.13	-.09	-.18	.01	.01	.07	-.09
ST.E.	.10	.10	.10	.10	.11	.11	.11	.11	.11	.11	.11	.11
13- 24	-.04	.06	-.11	.04	.03	.01	.05	.03	-.06	.11	.02	-.02
ST.E.	.11	.11	.11	.11	.11	.11	.11	.11	.11	.12	.12	.12

10.6 We generated $\{a_t\}$ and $\{b_t\}$ as independent N(0,1) random variables. The random walks were calculated recursively, starting with $y_1 = a_1$ and $x_1 = b_1$; the first 500 realizations were omitted in order to exclude any effect of the starting values. The results for series of length n = 50 are shown below. In 60 percent of the cases (6 out of 10), the regression slope was significant at the 0.05 level; the average R^2 was 0.14.

Estimate	Std.Error	t-ratio	prob-value	R**2
0.4572	0.2584	1.77	0.083	0.061
-0.15913	0.05370	-2.96	0.005	0.155
-0.47148	0.09568	-4.93	0.000	0.336
0.04544	0.05243	0.87	0.390	0.015
0.0509	0.1119	0.45	0.651	0.004
0.3334	0.1002	3.33	0.002	0.187
-0.4025	0.1223	-3.29	0.002	0.184
0.3952	0.1197	3.30	0.002	0.184
-0.18640	0.08463	-2.20	0.032	0.185
-0.1219	0.1221	-1.00	0.323	0.092

Different random variables were used in the simulation for the series of length n = 100. We find a significant relationship in 50 percent of the cases (5 of 10), even though such a relationship should occur in only 5 percent (significance level) of the cases. The average R^2 was 0.08.

Estimate	Std.Error	t-ratio	prob-value	R**2
0.01853	0.08113	0.23	0.820	0.001
-0.08713	0.04945	-1.76	0.081	0.031
0.46433	0.08420	5.51	0.000	0.237
-0.2079	0.1764	-1.18	0.241	0.014
-0.1564	0.1118	-1.40	0.165	0.020
0.13184	0.03684	3.58	0.001	0.116
0.10329	0.04375	2.36	0.020	0.054
0.55219	0.08888	6.21	0.000	0.283
-0.1201	0.1369	-0.88	0.383	0.008
0.1919	0.1299	1.48	0.143	0.022

These results show the problem of spurious relationships when regressing two independent autocorrelated series.

10.7 (a) Regression results for each of the four products are shown below. The coefficients of determination are larger than 50 percent. For some products one or the other regressor can be omitted. The independence assumption of the errors is violated in the regressions for products 2 and 4. In these cases the Durbin-Watson statistics are considerably smaller than 2, indicating positive lag 1 autocorrelation.

Product 1:

The regression equation is
Product1 = 26.7 + 3.87 Chemicals(Index) - 0.097 Industrial Equipment (Index)

Predictor	Coef	SE Coef	T	P
Constant	26.67	71.45	0.37	0.711
Chemical	3.8689	0.9406	4.11	0.000
Industrial	-0.0970	0.5528	-0.18	0.862

S = 21.79 R-Sq = 50.7% R-Sq(adj) = 47.7%

Durbin-Watson statistic = 2.17

The regression equation is
Product1 = 27.0 + 3.75 Chemicals(Index)

Predictor	Coef	SE Coef	T	P
Constant	26.97	70.38	0.38	0.704
Chemical	3.7502	0.6438	5.82	0.000

S = 21.47 R-Sq = 50.7% R-Sq(adj) = 49.2%

Durbin-Watson statistic = 2.18

Product 2:

The regression equation is
Product2 = - 44.6 + 0.217 Chemicals(Index) + 0.281 Industrial Equipment (Index)

Predictor	Coef	SE Coef	T	P
Constant	-44.55	11.23	-3.97	0.000
Chemical	0.2171	0.1479	1.47	0.152
Industrial	0.28123	0.08691	3.24	0.003

S = 3.426 R-Sq = 55.7% R-Sq(adj) = 53.0%

Durbin-Watson statistic = 1.09

The regression equation is
Product2 = - 32.8 + 0.373 Industrial Equipment (Index)

Predictor	Coef	SE Coef	T	P
Constant	-32.828	8.036	-4.09	0.000
Industrial	0.37300	0.06143	6.07	0.000

S = 3.485 R-Sq = 52.8% R-Sq(adj) = 51.3%

Durbin-Watson statistic = 1.03

Product 3:

```
The regression equation is
Product3 = - 315 + 2.06 Chemicals(Index) + 2.69 Industrial Equipment
(Index)

Predictor        Coef     SE Coef          T        P
Constant      -315.02       58.32      -5.40    0.000
Chemical       2.0556      0.7678       2.68    0.012
Industrial     2.6905      0.4513       5.96    0.000

S = 17.79       R-Sq = 81.0%     R-Sq(adj) = 79.8%

Durbin-Watson statistic = 1.51
```

Product 4:

```
The regression equation is
Product4 = - 61.1 + 0.669 Chemicals(Index) + 0.178 Industrial
Equipment (Index)

Predictor        Coef     SE Coef          T        P
Constant       -61.09       16.17      -3.78    0.001
Chemical       0.6695      0.2129       3.14    0.004
Industrial     0.1783      0.1251       1.42    0.164

S = 4.932       R-Sq = 54.3%     R-Sq(adj) = 51.5%

Durbin-Watson statistic = 0.83


The regression equation is
Product4 = - 61.7 + 0.888 Chemicals(Index)

Predictor        Coef     SE Coef          T        P
Constant       -61.65       16.42      -3.76    0.001
Chemical       0.8876      0.1502       5.91    0.000

S = 5.008       R-Sq = 51.4%     R-Sq(adj) = 49.9%

Durbin-Watson statistic = 0.84

Autocorrelations of the residuals
   0.526111   0.545286   0.317164   0.321774   0.213099  -0.025736
```

(b) None of the contemporaneous regressions in (a) are suitable for prediction purposes, as the indexes of future chemical and industrial production are not available. For prediction purposes one must find models that explain current sales as functions of previous values of the regressors.

We use the first four lags of each of the two explanatory variables (we believe that higher lags are probably not justified), and start our model search with the following

eight regressors: $Chem_{t-1}, Chem_{t-2}, Chem_{t-3}, Chem_{t-4}$ and $Ind_{t-1}, Ind_{t-2}, Ind_{t-3}, Ind_{t-4}$. Stepwise regression (see Chapter 7) is used to decide on the significant regressors. The results are shown below. The R-square from these regressions are quite similar to those from the contemporaneous regressions (the R-square of the lag regression for product 1 is lower), and we still have problems with autocorrelation, mostly for product 4. The 95 percent margins for the prediction error are at least $\pm 2s$. For product 2, for example, this amounts to $\pm 2(2.944) \approx \pm 6$. Judging from the past sales history of product 2, this indicates considerable uncertainty.
Lagged values of sales could also be incorporated into the regressions.

<u>Product 1:</u>

```
The regression equation is
Product1 = 296 + 3.26 ChemLag1 - 1.95 ChemLag4

31 cases used 4 cases contain missing values

Predictor        Coef     SE Coef          T        P
Constant        295.7       104.1       2.84    0.008
ChemLag1       3.2578      0.9092       3.58    0.001
ChemLag4      -1.9503      0.9061      -2.15    0.040

S = 25.71       R-Sq = 32.0%     R-Sq(adj) = 27.2%

Durbin-Watson statistic = 1.95
```

<u>Product 2:</u>

```
The regression equation is
Product2 = - 33.4 + 0.218 ChemLag1 + 0.600 ChemLag2 - 0.301 IndLag4

31 cases used 4 cases contain missing values

Predictor        Coef     SE Coef          T        P
Constant       -33.44       11.56      -2.89    0.007
ChemLag1       0.2183      0.1999       1.09    0.284
ChemLag2       0.5995      0.2120       2.83    0.009
IndLag4      -0.30113     0.07209      -4.18    0.000

S = 2.944       R-Sq = 70.0%     R-Sq(adj) = 66.7%

Durbin-Watson statistic = 1.46
```

<u>Product 3:</u>

```
The regression equation is
Product3 = - 283 + 2.47 ChemLag1 + 2.12 IndLag1

34 cases used 1 cases contain missing values
```

```
Predictor        Coef     SE Coef          T        P
Constant      -283.45       67.15      -4.22    0.000
ChemLag1       2.4658      0.9234       2.67    0.012
IndLag1        2.1152      0.5621       3.76    0.001

S = 20.48       R-Sq = 72.6%     R-Sq(adj) = 70.8%

Durbin-Watson statistic = 1.48
```

Product 4:

```
The regression equation is
Product3 = - 290 + 5.06 ChemLag1

34 cases used 1 cases contain missing values

Predictor        Coef     SE Coef          T        P
Constant      -289.87       79.75      -3.63    0.001
ChemLag1       5.0608      0.7296       6.94    0.000

S = 24.33       R-Sq = 60.1%     R-Sq(adj) = 58.8%

Durbin-Watson statistic = 1.07
```

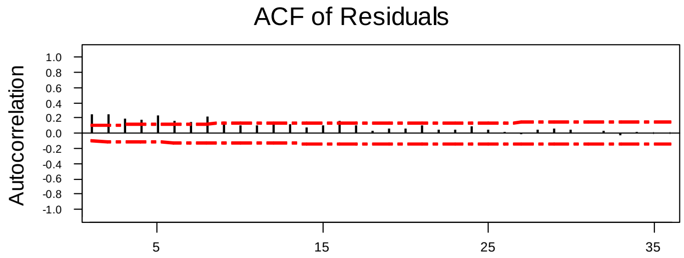
10.8 The autocorrelation function of the residuals in the regression model (brand P)

$$\ln \text{SalesP12}_t = \beta_0 + \beta_1 \ln \text{PriceP6}_t + \beta_2 \ln \text{PriceP12}_t + \beta_3 \ln \text{PriceP24}_t + \varepsilon_t$$

is shown below. When calculating the autocorrelations we had to omit a few weeks with missing observations. This affected the spacing of the observations, but this issue is ignored here. The standard error of the autocorrelations is about 0.05 . The autocorrelations decay very slowly and indicate nonstationarity. The Durbin-Watson statistic (DW = 1.49) indicates unacceptable positive lag 1 autocorrelation.

```
The regression equation is
lnsalesP12 = - 3.74 + 0.921 lnPriceP6 - 7.24 lnPriceP12 + 2.92
lnPriceP24

Predictor        Coef     SE Coef          T        P
Constant       -3.740       1.598      -2.34    0.020
LnPriceP6      0.9205      0.1603       5.74    0.000
LnPriceP12    -7.2420      0.3040     -23.82    0.000
LnPriceP24     2.9233      0.2895      10.10    0.000

S = 0.7338      R-Sq = 63.0%     R-Sq(adj) = 62.7%

Analysis of Variance

Source           DF          SS          MS        F        P
Regression        3      347.92      115.97   215.40    0.000
Residual Error  380      204.59        0.54
Total           383      552.51
```

```
Durbin-Watson statistic = 1.49
```



Lag	Corr	T	LBQ	Lag	Corr	T	LBQ	Lag	Corr	T	LBQ	Lag	Corr	T	LBQ
1	0.24	4.75	22.74	10	0.11	1.58	144.32	19	0.06	0.88	181.90	28	0.04	0.52	194.31
2	0.25	4.68	47.48	11	0.10	1.51	148.42	20	0.06	0.84	183.33	29	0.06	0.77	195.57
3	0.19	3.33	61.44	12	0.11	1.64	153.35	21	0.10	1.35	187.07	30	0.04	0.52	196.17
4	0.18	3.09	74.20	13	0.12	1.74	158.94	22	0.05	0.69	188.07	31	0.01	0.08	196.18
5	0.23	3.88	95.44	14	0.08	1.17	161.52	23	0.04	0.63	188.89	32	0.03	0.45	196.62
6	0.17	2.69	106.44	15	0.10	1.42	165.34	24	0.10	1.33	192.62	33	-0.03	-0.35	196.88
7	0.14	2.22	114.27	16	0.16	2.36	176.08	25	0.05	0.65	193.53	34	0.01	0.13	196.92
8	0.21	3.32	132.27	17	0.10	1.37	179.84	26	0.02	0.24	193.66	35	-0.00	-0.02	196.92
9	0.14	2.10	139.89	18	0.03	0.49	180.32	27	-0.01	-0.18	193.73	36	0.00	0.03	196.92

First differences of the residuals (not shown here) are stationary, with an autocorrelation function that shows a single large spike at lag 1. This suggests the noisy random walk (or ARIMA(0,1,1)), $(1-\mathrm{B})\varepsilon_t = (1-\theta\mathrm{B})a_t$, as an appropriate error model. Combining this with the previous regression leads to the model

$$\ln \mathrm{SalesP12}_t = \beta_1 \ln \mathrm{PriceP6}_t + \beta_2 \ln \mathrm{PriceP12}_t + \beta_3 \ln \mathrm{PriceP24}_t + \frac{1-\theta\mathrm{B}}{1-\mathrm{B}} a_t$$

or,

$$(1-\mathrm{B})[\ln \mathrm{SalesP12}_t] = \beta_1 (1-\mathrm{B})[\ln \mathrm{PriceP6}_t] + \beta_2 (1-\mathrm{B})[\ln \mathrm{PriceP12}_t] + \beta_3 (1-\mathrm{B})[\ln \mathrm{PriceP24}_t] + (1-\theta\mathrm{B})a_t \ .$$

Because of differencing we lose the ability to estimate the intercept β_0. The SCA estimation results are given below:

```
SUMMARY FOR UNIVARIATE TIME SERIES MODEL --    M1
 ----------------------------------------------------------------------
 VARIABLE     TYPE OF     ORIGINAL      DIFFERENCING
           VARIABLE    OR CENTERED
                                               1
 LNSalesP12     RANDOM      ORIGINAL       (1-B   )
                                               1
  LNPriceP6     RANDOM      ORIGINAL       (1-B   )
                                               1
 LNPriceP12     RANDOM      ORIGINAL       (1-B   )
                                               1
 LNPriceP24     RANDOM      ORIGINAL       (1-B   )
 ----------------------------------------------------------------------

  PARAMETER    VARIABLE  NUM./  FACTOR  ORDER   CONS-      VALUE      STD     T
    LABEL        NAME    DENOM.                TRAINT                ERROR   VALUE
```

```
  1    B1  LNPriceP6   NUM.      1      0     NONE     1.2561      .1500   8.37
  2    B2 LNPriceP12   NUM.      1      0     NONE    -6.6402      .3054 -21.74
  3    B3 LNPriceP24   NUM.      1      0     NONE     3.2115      .2677  12.00
  4 THETA              MA        1      1     NONE      .8644      .0250  34.62


 EFFECTIVE NUMBER OF OBSERVATIONS . .          383
 R-SQUARE . . . . . . . . . . . . . .        0.694
 RESIDUAL STANDARD ERROR. . . . . . .  0.663259E+00

AUTOCORRELATIONS OF RESIDUALS

  1- 12    -.00 -.01 -.05 -.03  .05 -.01 -.02  .08  .01 -.04 -.03  .01
  ST.E.     .05  .05  .05  .05  .05  .05  .05  .05  .05  .05  .05  .05

 13- 24     .01 -.02  .01  .11  .03 -.06 -.01 -.01  .01 -.01 -.02  .06
  ST.E.     .05  .05  .05  .05  .05  .05  .05  .05  .05  .05  .05  .05
```

The estimate of θ is close to one. This model is equivalent to one that relates differences of log sales to differences of log prices, with a moving average error component that is close to one. Recall that first differences of logs are equivalent to percentage changes.

The "design" of the price data is interesting, as there are periods where prices are rather flat. Look at the time series graph of (log) prices. One notices a certain "industry price" which stores use as the base when reducing their prices. Every once in a while the industry price changes. One could argue that it is not the actual price, but the "un-anticipated" price that matters and affects sales. One could measure the "un-anticipated" price component by considering the difference between the current price, p_t, and the exponentially weighted average of past prices. That is, one could consider

$$p_t - (1-\alpha)[p_{t-1} + \alpha p_{t-2} + \alpha^2 p_{t-3} + ...] = p_t - (1-\alpha)\frac{B}{1-\alpha B}p_t = \frac{1-B}{1-\alpha B}p_t$$

as the relevant regressor variable. The parameter α determines how quickly price information is discounted. [Here B is the backshift operator. Check that the left hand side of the above expression can be written this way. For simplicity of exposition we have considered a single price series.]

Regressing y_t on the un-anticipated price component leads to the model

$$y_t = \beta_0 + \beta_1[(1-B)/(1-\alpha B)]p_t + ... + \varepsilon_t$$

or,

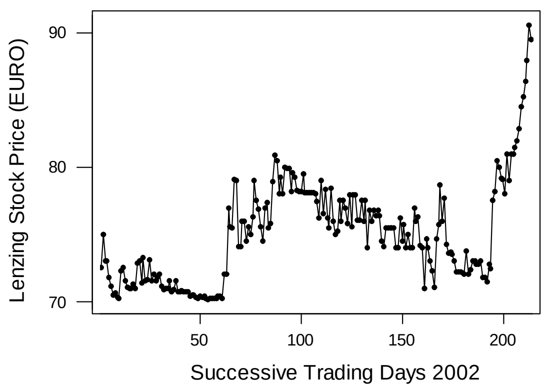
$$(1-\alpha B)y_t = \beta_0^* + \beta_1(1-B)p_t + ... + (1-\alpha B)\varepsilon_t$$

Note that this derivation assumes that α is the same for all three price series. The estimation results for this model are shown below. The estimate of α is close to one. In essence, this model goes back to the model with differences in all variables (response as well as regressor variables) and a moving average parameter that is close to one. The estimates of the regression coefficients (1.11, -6.57, 2.95) are similar to the coefficients in the earlier regression time series model (1.26, -6.64, 3.21).

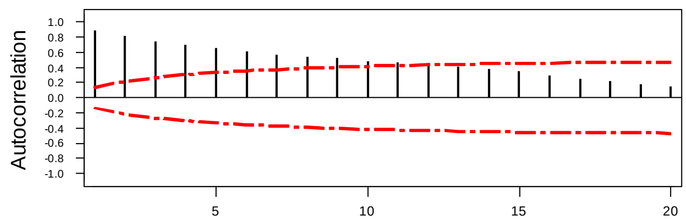
```
----------------------------------------------------------------------
 VARIABLE   TYPE OF    ORIGINAL     DIFFERENCING
           VARIABLE   OR CENTERED

 LNSalesP12    RANDOM     ORIGINAL     NONE
 LNSalesP12    RANDOM     ORIGINAL     NONE
                                           1
  LNPriceP6    RANDOM     ORIGINAL     (1-B  )
                                           1
 LNPriceP12    RANDOM     ORIGINAL     (1-B  )
                                           1
 LNPriceP24    RANDOM     ORIGINAL     (1-B  )
 ----------------------------------------------------------------------

  PARAMETER   VARIABLE  NUM./  FACTOR  ORDER   CONS-     VALUE      STD     T
    LABEL       NAME    DENOM.                 TRAINT               ERROR  VALUE

   1  CNST             CNST      1       0     NONE      .6756      .1573   4.30
   2 THETA  LNSalesP12   NUM.    1       1    EQ 01      .9214      .0162  56.70
   3   B1    LNPriceP6   NUM.    1       0     NONE     1.1090      .1778   6.24
   4   B2   LNPriceP12   NUM.    1       0     NONE    -6.5664      .3564 -18.42
   5   B3   LNPriceP24   NUM.    1       0     NONE     2.9520      .3159   9.35
 *** THETA  LNSalesP12    MA     1       1    EQ 01      .9214      .0162  56.70


 EFFECTIVE NUMBER OF OBSERVATIONS . .            383
 R-SQUARE . . . . . . . . . . . . . .          0.581
 RESIDUAL STANDARD ERROR. . . . . . .  0.776764E+00
```

10.9 The time series graph shows that the level of the series changes over time. The series is not stationary. Stock price data are usually nonstationary, with changing levels and locally changing trends. Note that we treat the time series observations as equally spaced, despite the fact that there is no trading on weekends and holidays. The autocorrelation function of the series is slow to die down. This is yet another indication of nonstationary.

Exercise 10.9: Lenzing Stock Prices



Autocorrelation Function for Lenzing Stock

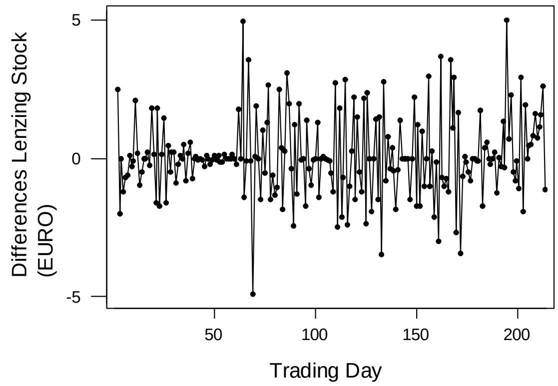


Lag	Corr	T	LBQ	Lag	Corr	T	LBQ	Lag	Corr	T	LBQ
1	0.89	13.02	171.96	8	0.54	2.73	872.83	15	0.35	1.50	1182.52
2	0.82	7.48	319.43	9	0.52	2.56	934.43	16	0.30	1.29	1203.56
3	0.75	5.56	444.15	10	0.49	2.32	988.64	17	0.25	1.08	1218.60
4	0.71	4.58	553.68	11	0.47	2.19	1039.62	18	0.21	0.91	1229.47
5	0.66	3.91	649.53	12	0.44	1.98	1083.35	19	0.18	0.75	1237.05
6	0.61	3.40	732.73	13	0.41	1.81	1121.25	20	0.15	0.62	1242.26
7	0.58	3.05	807.56	14	0.38	1.67	1154.60				

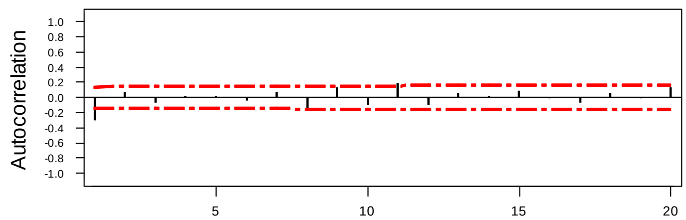
First differences of the series have a constant level and are stationary. Autocorrelations of first differences die down rapidly. In fact, only the lag one autocorrelation exceeds twice the standard error, $1/\sqrt{213} = 0.07$. Adjacent changes of stock prices are correlated. Note that also, the lag 11 autocorrelation exceeds twice the standard error. However, we doubt that changes 11 steps apart are really correlated, and we attribute this autocorrelation to chance.

The time series graph of first differences shows periods where there is more (and less) variability (also called volatility). Time series models that incorporate components for changing variability (ARCH and GARCH models) are studied in the finance literature.

Exercise 10.9: Differences of Lenzing Stock Prices

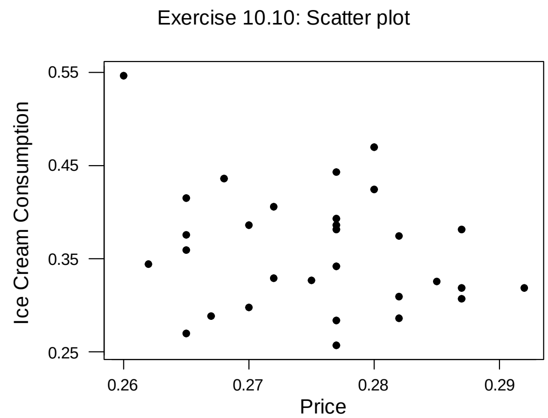


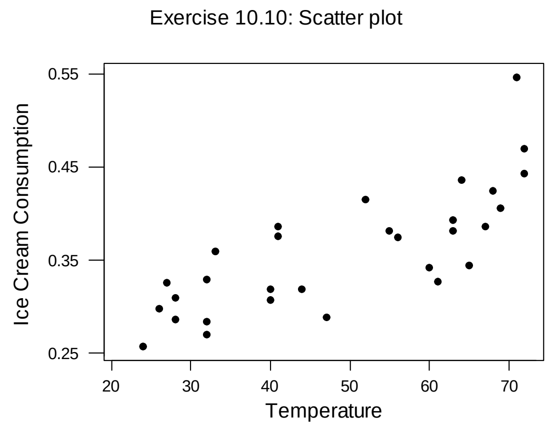
Autocorrelation Function for Differences of Lenzing Stock



Lag	Corr	T	LBQ	Lag	Corr	T	LBQ	Lag	Corr	T	LBQ
1	-0.31	-4.47	20.30	8	-0.14	-1.87	28.76	15	0.08	1.02	48.32
2	0.07	0.94	21.36	9	0.13	1.64	32.34	16	-0.02	-0.26	48.42
3	-0.08	-1.04	22.68	10	-0.11	-1.39	34.97	17	-0.08	-0.94	49.80
4	0.01	0.18	22.72	11	0.20	2.49	43.70	18	0.06	0.77	50.76
5	0.01	0.18	22.76	12	-0.10	-1.24	45.99	19	-0.01	-0.13	50.78
6	-0.04	-0.59	23.19	13	0.05	0.66	46.67	20	0.14	1.68	55.37
7	0.07	0.92	24.27	14	0.01	0.14	46.70				

10.10 Scatter plots of ice cream consumption on price, family income, and temperature, and results of fitting the regression model $\text{Cons}_t = \beta_0 + \beta_1 \text{Price}_t + \beta_2 \text{Inc}_t + \text{Temp}_t + \varepsilon_t$ are shown below. The Durbin-Watson statistic is much smaller than the desired value 2 and unacceptable. The small value of the Durbin-Watson statistic indicates positive lag one autocorrelation. The first six autocorrelations of the residuals are also shown. Especially the lag one autocorrelation (r_1 = 0.32) is relatively large when compared to its standard error $1/\sqrt{30} = 0.18$.


Exercise 10.10: Scatter plot
Ice Cream Consumption
0.55
0.45
0.35
0.25
0.26
0.27
0.28
0.29
Price


Exercise 10.10: Scatter plot
Ice Cream Consumption
0.55
0.45
0.35
0.25
20
30
40
50
60
70
Temperature


Exercise 10.10: Scatter plot
Ice Cream Consumption
0.55
0.45
0.35
0.25
75
85
95
Family Income

```
The regression equation is
Consumption = 0.197 - 1.04 Price + 0.00331 Income + 0.00346
Temperature

Predictor        Coef     SE Coef         T        P
Constant       0.1973      0.2702      0.73    0.472
Price         -1.0444      0.8344     -1.25    0.222
Income       0.003308    0.001171      2.82    0.009
Temperature 0.0034584   0.0004455      7.76    0.000

S = 0.03683     R-Sq = 71.9%     R-Sq(adj) = 68.7%

Analysis of Variance

Source            DF          SS          MS         F        P
Regression         3    0.090251    0.030084     22.17    0.000
Residual Error    26    0.035273    0.001357
Total             29    0.125523

Durbin-Watson statistic = 1.02

Autocorrelations of Residuals

   0.329772   0.036248   0.011063  -0.093395  -0.318641  -0.205802
```

The errors in this regression are not independent, and the error model needs to be revised. We consider two different error models: a first-order moving average and a first-order autoregressive error model. Note that in the regression with independent errors the coefficient for price is not significant. However, we keep this variable in the model as the significance may have been affected by the correlations in the errors. If it turns out that this coefficient is still insignificant, it can be removed at a later stage.

Estimation results for the two models are shown below. We use SCA to carry out the estimation. Alternatively, one can use EVIEWS. The residuals of the revised models are uncorrelated. The regression coefficients for income and temperature are significant (t-ratios exceed two). Income and temperature have positive regression coefficients; ice cream sales increase with increasing income and rising temperature. The coefficient of price is negative and not very significant (t-ratios of -1.77 and -1.18, respectively).

MA(1): $\text{Cons}_t = \beta_0 + \beta_1 \text{Price}_t + \beta_2 \text{Inc}_t + \beta_3 \text{Temp}_t + (1-\theta B)a_t$

PARAMETER LABEL		VARIABLE NAME	NUM./ DENOM.	FACTOR	ORDER	CONS-TRAINT	VALUE	STD ERROR	T VALUE
1	B0		CNST	1	0	NONE	.3287	.2661	1.24
2	B1	PRICE	NUM.	1	0	NONE	-1.3886	.7829	-1.77
3	B2	INCOME	NUM.	1	0	NONE	.0029	.0014	2.15
4	B3	TEMP	NUM.	1	0	NONE	.0034	.0005	6.64
5	THETA	ICE	MA	1	1	NONE	-.5031	.1760	-2.86

```
EFFECTIVE NUMBER OF OBSERVATIONS . .           30
 R-SQUARE . . . . . . . . . . . . . . .       0.771
 RESIDUAL STANDARD ERROR. . . . . . .  0.309303E-01

 AUTOCORRELATIONS OF RESIDUALS

   1- 12     .02  .06 -.01  .02 -.30  .01 -.14 -.13 -.01 -.17 -.13  .07
   ST.E.     .18  .18  .18  .18  .18  .20  .20  .20  .21  .21  .21  .21

  13- 24     .32  .10  .02  .07  .13 -.15 -.04  .05 -.03 -.18  .01 -.23
   ST.E.     .21  .23  .23  .23  .23  .23  .24  .24  .24  .24  .24  .24
```

AR(1): $\text{Cons}_t = \beta_0 + \beta_1 \text{Price}_t + \beta_2 \text{Inc}_t + \beta_3 \text{Temp}_t + \dfrac{1}{(1-\phi B)} a_t$

PARAMETER LABEL		VARIABLE NAME	NUM./ DENOM.	FACTOR	ORDER	CONS- TRAINT	VALUE	STD ERROR	T VALUE
1	B0		CNST	1	0	NONE	.1495	.2697	.55
2	B1	PRICE	NUM.	1	0	NONE	-.8889	.7532	-1.18
3	B2	INCOME	NUM.	1	0	NONE	.0033	.0014	2.33
4	B3	TEMP	NUM.	1	0	NONE	.0035	.0005	6.57
5	PHI	ICE	D-AR	1	1	NONE	.4016	.1866	2.15

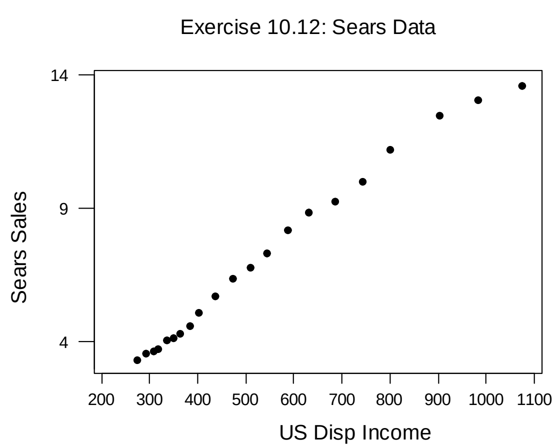
```
 EFFECTIVE NUMBER OF OBSERVATIONS . .          29
 R-SQUARE . . . . . . . . . . . . . . .       0.790
 RESIDUAL STANDARD ERROR. . . . . . .  0.296282E-01

AUTOCORRELATIONS OF RESIDUALS

   1- 12     .09 -.11 -.02  .04 -.15  .08 -.10 -.09  .01 -.29 -.24  .09
   ST.E.     .19  .19  .19  .19  .19  .19  .19  .20  .20  .20  .21  .22

  13- 24     .38  .07 -.01  .00  .14 -.06  .02  .06  .03 -.09 -.12 -.20
   ST.E.     .22  .24  .24  .24  .24  .25  .25  .25  .25  .25  .25  .25
```

10.11 The scatter plot of lake levels against sunspots and the results of fitting the regression $\text{LakeLevel}_t = \beta_0 + \beta_1 \text{Sunspots}_t + \varepsilon_t$ are shown below. The Durbin-Watson statistic (1.71) and the autocorrelations of the residuals (with standard error $1/\sqrt{20} = 0.22$) indicate that there is no problem with serial correlation. The errors can be assumed independent.

```
The regression equation is
LakeLevel = - 8.04 + 0.413 Sunspots

Predictor        Coef     SE Coef          T        P
Constant       -8.042       2.556      -3.15    0.006
Sunspots      0.41281     0.05275       7.83    0.000

S = 6.466       R-Sq = 77.3%     R-Sq(adj) = 76.0%
```

```
Analysis of Variance

Source            DF         SS          MS         F        P
Regression         1     2560.4      2560.4     61.24    0.000
Residual Error    18      752.5        41.8
Total             19     3313.0

Unusual Observations
Obs  Sunspots   LakeLeve         Fit      SE Fit   Residual   St Resid
  5        54      29.00       14.25        1.62      14.75      2.36R
 16       104      35.00       34.89        3.67       0.11      0.02 X

R denotes an observation with a large standardized residual
X denotes an observation whose X value gives it large influence.

Durbin-Watson statistic = 1.71

Autocorrelations of residuals
   0.100203   0.027064   0.284582   0.100791
```

10.12 (a) The scatter plots of sales on disposable income and the results of fitting the regression model $\text{Sales}_t = \beta_0 + \beta_1 \text{Income}_t + \varepsilon_t$ are shown below. The Durbin-Watson statistics is much smaller than the desired value 2, and is unacceptable. The small value of the Durbin-Watson statistic indicates positive lag one autocorrelation. The first four autocorrelations of the residuals are also shown. The lag one autocorrelation ($r_1 = 0.48$) exceeds twice its standard error $1/\sqrt{21} = 0.22$.




```
The regression equation is
Sales = - 0.524 + 0.0140 Income

Predictor        Coef     SE Coef          T        P
Constant      -0.5243      0.1884      -2.78    0.012
Income      0.0140496   0.0003185      44.11    0.000
```

```
S = 0.3435      R-Sq = 99.0%     R-Sq(adj) = 99.0%

Analysis of Variance

Source            DF          SS          MS         F        P
Regression         1      229.60      229.60   1945.85    0.000
Residual Error    19        2.24        0.12
Total             20      231.85

Durbin-Watson statistic = 0.63

Autocorrelations of residuals

   0.478152   0.075695   0.060663   0.200269
```

(b) The errors in the regression are not independent, and the error model needs to be revised. We consider a noisy random walk (the ARIMA(0,1,1)) model

$$\text{Sales}_t = \beta_0 + \beta_1 \text{Income}_t + \frac{1-\theta B}{(1-B)} a_t \ , \quad \text{or}$$

$$(1-B)\text{Sales}_t = \beta_1 (1-B)\text{Income}_t + (1-\theta B) a_t$$

Because of the differencing operation it is no longer possible to estimate the intercept β_0 in the earlier regression model. The SCA estimation results show that this model fits much better. The residuals are uncorrelated; especially the lag one autocorrelation is much smaller. Note that with a small data set such as this (n = 21), various other noise models could be considered to approximate the autocorrelation of the errors in the regression model in (a).

```
----------------------------------------------------------------------
 VARIABLE   TYPE OF    ORIGINAL     DIFFERENCING
           VARIABLE   OR CENTERED
                                         1
  SALES     RANDOM     ORIGINAL     (1-B  )
                                         1
   INC      RANDOM     ORIGINAL     (1-B  )
 ----------------------------------------------------------------------

  PARAMETER   VARIABLE  NUM./  FACTOR  ORDER   CONS-     VALUE      STD     T
    LABEL       NAME    DENOM.                 TRAINT               ERROR  VALUE

   1    B1       INC     NUM.      1      0     NONE      .0107     .0014   7.77
   2  THETA     SALES     MA       1      1     NONE     -.7308     .1504  -4.86

 EFFECTIVE NUMBER OF OBSERVATIONS . .           20
 R-SQUARE . . . . . . . . . . . . . . .       0.997
 RESIDUAL STANDARD ERROR. . . . . . .  0.190304E+00

AUTOCORRELATIONS OF RESIDUALS

   1- 6      .08 -.24 -.27  .31  .11 -.15
   ST.E.     .22  .23  .24  .25  .27  .27
```

(c) For $\theta = 0$ (which, however, is not indicated from the data), the model in (b) simplifies to a regression of $ChangeSales_t = (Sales_t - Sales_{t-1})$ on changes in disposable income $ChangeIncome_t = (Income_t - Income_{t-1})$. The results of this regression are given below. The Durbin-Watson statistic is still much smaller than the desired value 2, and the first four autocorrelations of the residuals are barely within two standard errors ($1/\sqrt{20} = 0.22$). The results indicate that a moving average component (and hence the ARIMA(0,1,1) model in part (b)) are needed.

```
The regression equation is
ChangeSales = 0.149 + 0.00916 ChangeIncome

20 cases used 1 cases contain missing values

Predictor        Coef     SE Coef          T        P
Constant      0.14892     0.09770       1.52    0.145
ChangeInc    0.009155    0.002034       4.50    0.000

S = 0.2397      R-Sq = 53.0%     R-Sq(adj) = 50.3%

Analysis of Variance

Source            DF          SS          MS         F        P
Regression         1      1.1646      1.1646     20.27    0.000
Residual Error    18      1.0344      0.0575
Total             19      2.1990

Durbin-Watson statistic = 1.12
Autocorrelation of residuals
   0.332880  -0.398696  -0.191203   0.333627
```

(d) The results of the regression of $RelChangeSales_t = (Sales_t - Sales_{t-1})/Sales_{t-1} \cong \ln(Sales_t) - \ln(Sales_{t-1})$ on relative changes in disposable income, $RelChangeIncome_t = (Income_t - Income_{t-1})/Income_{t-1} \cong \ln(Income_t) - \ln(Income_{t-1})$ are shown below. The results are similar to those in part (c) of the exercise.

```
The regression equation is
RelChaSales = 0.0219 + 0.732 RelChaInc

20 cases used 1 cases contain missing values

Predictor        Coef     SE Coef          T        P
Constant      0.02187     0.02449       0.89    0.384
RelChaIn       0.7322      0.3290       2.23    0.039

S = 0.03219     R-Sq = 21.6%     R-Sq(adj) = 17.2%
Analysis of Variance

Source            DF          SS          MS         F        P
Regression         1    0.005131    0.005131      4.95    0.039
Residual Error    18    0.018648    0.001036
Total             19    0.023778
```

```
Durbin-Watson statistic = 1.30

Autocorrelations of residuals
   0.284050  -0.322172  -0.086133   0.319946
```

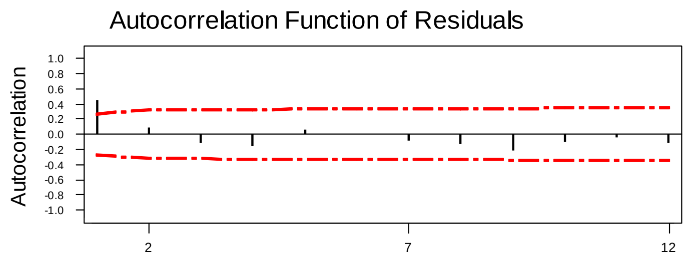
(e) The model in part (b) gives a good description of the data.

10.13 (a) Results of the regression

$$FTEShares_t = \beta_0 + \beta_1 CarProd_{t-6} + \beta_2 FTECom_{t-7} + \varepsilon_t$$

are given below. The Durbin-Watson statistics is much smaller than the desired value 2 and is unacceptable. The small value of the Durbin-Watson statistic indicates positive lag one autocorrelation. The autocorrelation function of the residuals indicates significant autocorrelations, especially at lag 1 ($r_1 = 0.45$, compared to its standard error $1/\sqrt{22} = 0.13$). The extremely significant estimates for lagged car production and lagged commodity index are surprising, because results in the finance literature indicate that stock prices are best predicted by the current value of the stock, but not by other economic variables.

```
The regression equation is
FTEShare = 595 + 0.000514 CarLag6 - 5.54 ComLag7

55 cases used 7 cases contain missing values

Predictor        Coef     SE Coef          T        P
Constant       594.51       60.65       9.80    0.000
CarLag6    0.00051422  0.00003406      15.10    0.000
ComLag7       -5.5439      0.6727      -8.24    0.000

S = 25.06       R-Sq = 88.2%     R-Sq(adj) = 87.8%

Analysis of Variance

Source            DF          SS          MS         F        P
Regression         2      244274      122137    194.46    0.000
Residual Error    52       32661         628
Total             54      276935

Durbin-Watson statistic = 0.87
```



Lag	Corr	T	LBQ	Lag	Corr	T	LBQ
1	0.45	3.36	11.94	8	-0.13	-0.80	16.84
2	0.09	0.57	12.42	9	-0.22	-1.31	20.19
3	-0.11	-0.69	13.16	10	-0.10	-0.59	20.92
4	-0.17	-1.04	14.91	11	-0.04	-0.24	21.05
5	0.05	0.32	15.08	12	-0.12	-0.71	22.17
6	0.01	0.04	15.08				
7	-0.09	-0.56	15.64				

(b) We consider the noisy random walk as a model for the errors, and fit the regression model

$$\text{FTEShares}_t = \beta_1 \text{Car Prod}_{t-6} + \beta_2 \text{FTECom}_{t-7} + \frac{1-\theta B}{1-B} a_t .$$

Because of the differencing operation, it is no longer possible to estimate the intercept β_0 of the earlier regression model.

VARIABLE	TYPE OF VARIABLE	ORIGINAL OR CENTERED	DIFFERENCING
FTESHARE	RANDOM	ORIGINAL	$(1-B^1)$
CAR	RANDOM	ORIGINAL	$(1-B^1)$
FTECOMM	RANDOM	ORIGINAL	$(1-B^1)$

PARAMETER LABEL		VARIABLE NAME	NUM./ DENOM.	FACTOR	ORDER	CONS-TRAINT	VALUE	STD ERROR	T VALUE
1	B1	CarProd	NUM.	1	6	NONE	.0001	.8107E-04	1.81
2	B2	FTECom	NUM.	1	7	NONE	-.6884	1.1833	-.58
3	THETA	FTEShares	MA	1	1	NONE	-.1468	.1417	-1.04

```
EFFECTIVE NUMBER OF OBSERVATIONS . .             54
R-SQUARE . . . . . . . . . . . . . .          0.951
RESIDUAL STANDARD ERROR. . . . . . .  0.180416E+02
```

AUTOCORRELATIONS OF RESIDUALS

1- 12	-.03	-.04	.00	-.12	.07	.02	-.15	-.00	-.32	.11	-.11	-.18
ST.E.	.14	.14	.14	.14	.14	.14	.14	.14	.14	.15	.16	.16
13- 24	.12	-.09	.03	.27	-.15	.09	.08	.10	-.01	-.17	.06	-.03
ST.E.	.16	.16	.16	.16	.17	.17	.18	.18	.18	.18	.18	.18

(c) The estimate of θ is not much different from zero. We set it zero and estimate the parameters in the regression model with random walk errors

$$\text{FTEShares}_t = \beta_1 \text{CarProd}_{t-6} + \beta_2 \text{FTECom}_{t-7} + \frac{1}{1-B} a_t \;.$$

This model is a regression of differences of the response on differences of the regressor variables,

$$\Delta\text{FTEShares}_t = \beta_1 \Delta\text{CarProd}_{t-6} + \beta_2 \Delta\text{FTECom}_{t-7} + a_t \,.$$

The results given below show that there is no autocorrelation in the residuals. The model passes all diagnostic checks. The intercept and the regressors are not statistically significant (p-values of 0.085 and 0.51), implying that the model for the FTE share index is given by the random walk

$$\Delta\text{FTEShares}_t = \text{FTEShares}_t - \text{FTEShares}_{t-1} = a_t \,.$$

This result is expected. The finance literature shows that in efficient markets stock prices follow random walks and changes in stock prices are unrelated to economic variables. The “significant” regression in part (a) was spurious, implied by the incorrect model for the error terms; see the discussion of spurious regression in Section 10.2.

```
The regression equation is
DiffShare = 3.71 +0.000144 DiffCarLag6 - 0.79 DiffCommLag7

54 cases used 8 cases contain missing values

Predictor        Coef     SE Coef          T        P
Constant        3.712       2.547       1.46    0.151
DiffCarPr  0.00014414  0.00008218       1.75    0.085
DiffComm       -0.786       1.175      -0.67    0.507

S = 18.35       R-Sq = 6.1%      R-Sq(adj) = 2.4%

Analysis of Variance

Source           DF          SS          MS         F        P
Regression        2      1107.6       553.8      1.64    0.203
Residual Error   51     17179.0       336.8
Total            53     18286.5

Durbin-Watson statistic = 1.72

Autocorrelations of residuals

   0.100215  -0.050381  -0.024561  -0.117140   0.070461   0.023664
  -0.147233  -0.070849  -0.317866   0.050094  -0.118589  -0.173089
```