QUEEN'S DUPONT BACK PAIN STUDY
Calibration of a Magnetic Tracking Device Using Locally Linear Fits

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INTRODUCTION

Magnetic tracking devices are commonly used in biomechanics and kinesiology. Use of such systems is restrained both by limitations in system accuracy when a large volume of operation is desired and by their inherent sensitivity to large metallic objects. Use of a long range transmitters can improve performance with regards to the first limitation, but fails to overcome the second (Day et al. 1998). In this abstract, a calibration method is developed and tested on a Polhemus Fastrak® with a long range transmitter.

PROCEDURES

An apparatus was constructed which facilitated placement of a sensor in multiple known positions and orientations with respect to the transmitter. It consisted of an upright framework and jig. The framework consisted of a rectangular sheet of Plexiglas® fitted vertically into a slotted wooden base. There were nine parallel slots on the base spaced 20cm apart. Holes were drilled into the Plexiglas sheet in a grid of five columns and eight rows with a 20 cm spacing. The jig was constructed in such a way that a total of 24 orientations could be reproduced in each hole (Day et al. 1998). For this study, data were collected in 24 orientations for each of 12 holes in 5 of the 9 slots. A total volume of 1.6 x 0.8 x 1.2 m was calibrated with a grid spacing of 40cm.

1440 data points were collected at 60 positions. For each point, the true position vector $\mathbf{x}=(x,y,z)$ and the true orientation $\mathbf{A_t}$ were known, and the observed position vector $\mathbf{x_o}$ and orientation matrix $\mathbf{A_o}$ could be calculated. As well, the observed errors $\mathbf{x_e}=\mathbf{x_o}-\mathbf{x}$ and $\mathbf{A_e}=\mathbf{A_o}\mathbf{A_t}^{-1}$ were calculated.

Locally linear models were fit to predict each of the 12 components of $\mathbf{x}_e$ and $\mathbf{A}_e$ based on the true position $\mathbf{x}$. Using these fits, the following steps were used to correct the data: *i*) the true position was estimated by solving for the position $\hat{\mathbf{x}}$ at which $\mathbf{x_o} = \hat{\mathbf{x}} + \hat{\mathbf{x}}_\mathbf{e}(\hat{\mathbf{x}})$ where $\hat{\mathbf{x}}_\mathbf{e}(\hat{\mathbf{x}})$ was the estimate of the position error if the true location were equal to $\hat{\mathbf{x}}$, *ii*) the rotation matrix estimate $\hat{\mathbf{A}}_\mathbf{e}$ was calculated by estimating each of its components at $\hat{\mathbf{x}}$, and calculating the nearest orthogonal matrix, *iii*) the true rotation matrix was estimated by $\hat{\mathbf{A}} = \hat{\mathbf{A}}_\mathbf{e}^{-1}\mathbf{A_o}$.

The scalar components of $\hat{\mathbf{x}}_\mathbf{e}$ and $\hat{\mathbf{A}}_\mathbf{e}$ were estimated at each point using a local linear regression. This linear regression was weighted both to create a Gaussian kernel centred at the point of interest ($\mathbf{x_o}$), and to account for increased measurement variance as the distance between the sensor and transmitter increased. The weights were calculated as follows: Length *n* vector $\mathbf{y}$ was created for each component of interest as well as vector $\mathbf{X}$, an *n* x 4 vector where row *i* consists of a weight constant and the known true position values *i.e.* (1, *xi*, *yi*, *zi*). The weight constants for the vector of weights $\mathbf{w}(\sigma)$ at point $\mathbf{x_o}$ were calculated for each point:

$$w_i(\sigma) = |\mathbf{x_i}|^{-\mathbf{p}} \exp\left(\frac{|\mathbf{x_i} - \mathbf{x_o}|^2}{2\sigma^2}\right)$$

where $\mathbf{x_i}$ is the true position for the calibration point i, $\mathbf{p}$ is a power variable used to compensate for distance related measurement variance, and σ is a tuning parameter of the Gaussian kernel. Values for p were chosen graphically for the dataset. σ was tuned so as to minimize cross validation residuals (Hastie & Tibrishirani, 1990). Using the calculated weights, the local linear regression fits the model:

$$y_i = \beta_1 + \beta_2 x_i + \beta_3 y_i + \beta_4 z_i + e_i$$

by solving

$$X^t W(\sigma) X\beta = X^t W(\sigma)\mathbf{y}$$

where $W(\sigma)$ is a square matrix with diagonal entries $w(\sigma)$. This can be solved directly using an LU decomposition (Press et al. 1986). Given the solution the predicted value at $\mathbf{x}_0$ is calculated as:

$$\hat{y}_0 = \hat{\beta}_1 + \hat{\beta}_2 x_0 + \hat{\beta}_3 y_0 + \hat{\beta}_4 z_0$$

and it is this equation which is used in calculating each component of $\hat{\mathbf{x}}_\mathbf{e}(\hat{\mathbf{x}})$.

RESULTS

Measurement errors and variance both increased as the sensor was moved further from the transmitter. The distortion of measurements tended to ‘bend’ or ‘warp’ the measurement space. This applied to both position and orientation measurements (Figure 1 and 2).

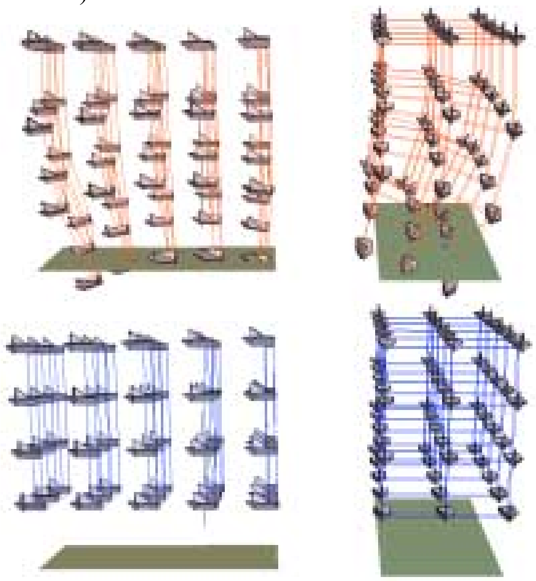

Figure 1 Measurement errors represented graphically in two orthogonal views both before (top) and after (bottom) calibration, Position errors are represented by the distortion of the grid while orientation errors are represented by the ‘tilt’ of the ‘sailboats’.

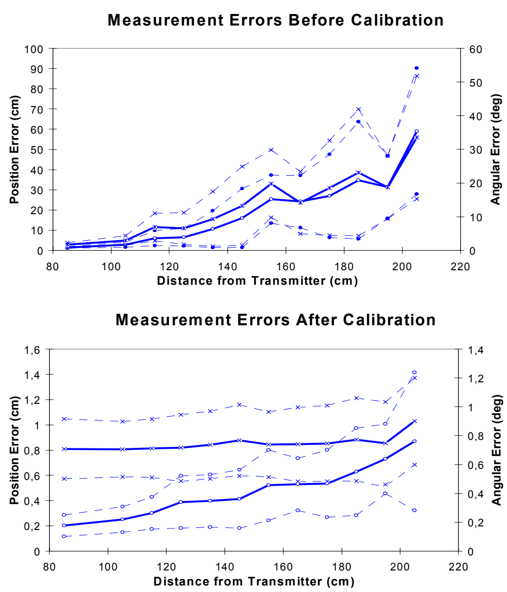

Figure 2 Average measurement errors (solid lines) and the standard deviation (dashed lines) are displayed for 10 cm intervals. Position errors are denoted by circles and angular errors by triangles.

Graphical examination of variance relations resulted in the following choices for **p**: 4 for the position measurements, 6 for diagonal rotation matrix entries and 1 for off diagonal rotation matrix entries. A 15 cm window size (σ) was chosen for the Gaussian kernel.

After calibration, average errors were reduced to less than 1cm for position measurements and less than 1 degree for orientation measurements (Figure 1 and 2).

DISCUSSION

Measurement errors due to the calibration frame and jig were approximately 0.5 cm and 0.8 degrees. This probably limited the performance of the calibration procedure.

Calibration measurements were taken in 24 orientations. This is a time consuming procedure. A concurrent evaluation of the system indicated that orientation dependant biases are small at distances less than 3m for the long range transmitter. Therefore, one orientation may yield sufficient results within this range (Day et al. 1998).

The calibration procedure was successful in reducing the magnitude of measurement errors to a level where the system could be useful for motion analysis of such tasks as lifting or gross analysis of gait.

REFERENCES

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