FM 2555A
Solutions to Chapter 4 – Practice Exercises

Problem [5/99]
Dividend discount model. Company Z's earnings and dividends per share are expected to grow indefinitely by 5% a year. If the next year's dividend is $10 and the market capitalisation rate is 8%, what is the current stock price?

SOLUTION:

$P_0 = \text{DIV}_1 / (r – g)$
$P_0 = \$10 / (.08 – .05)$
$P_0 = \$333.33$

Problem [6/100]
Company Z-prime is like Z in all respects save one: Its growth will stop after 4 years. In year 5 and afterward, it will pay out all earnings as dividends. What is Z-prime's stock price? Assume next year's EPS is $15.

SOLUTION:

$P_4 = \text{EPS}_5 / r$
$P_4 = [\text{EPS}_1 \times (1 + g_1)^3 \times (1 + g_2)] / r$
$P_4 = [\$15 \times (1 + .05)^3 \times (1 + 0)] / .08$
$P_4 = \$217.05$

Note that $15 is the EPS for year 1. The 5 percent growth rate stops after year 4, so the exponent for the first growth rate must be 3, (Year 4 – Year 1). There is no growth in year 5.

$P_0 = \text{DIV}_1 / (1 + r) + [\text{DIV}_1 \times (1 + g)] / (1 + r)^2 + [\text{DIV}_1 \times (1 + g)^2] / (1 + r)^3 + [\text{DIV}_1 \times (1 + g)^3] / (1 + r)^4 + P_4 / (1 + r)^4$
$P_0 = \$10 / 1.08 + (\$10 \times 1.05) / 1.08^2 + (\$10 \times 1.05^2) / 1.08^3 + (\$10 \times 1.05^3) / 1.08^4 + \$217.05 / 1.08^4$
$P_0 = \$195.06$

Problem [7/100]
If company Z (see Problem 5) were to distribute all its earnings, it could maintain a level dividend stream of $15 a share. How much is the market actually paying per share for growth opportunities?

SOLUTION:

$P_0 = \text{DIV}_1 / (r – g)$
$P_0 = \$10 / (.08 – .05)$
$P_0 = \$333.33$

$P_0 = \text{EPS}_1 / r + \text{PVGO}$
$\text{PVGO} = \$333.33 – \$15 / .08$
$\text{PVGO} = \$145.83$

Problem [8/100]

Dividend discount model. Consider three investors:

a. Mr Single invests for one year.
b. Ms Double invests for two years.
c. Mrs Triple invest for three years.

Assume each invests in company Z (see Problem 5). Show that each expects to earn a rate of return of 8% per year.

SOLUTION:

$DIV_1 = \$10$
$DIV_2 = DIV_1 \times (1 + g) = \$10 \times 1.05 = \$10.50$
$DIV_3 = DIV_2 \times (1 + g) = \$10.50 \times 1.05 = \$11.03$

$P_0 = DIV_1 / (r - g) = \$10 / (.08 - .05) = \$333.33$
$P_1 = P_0 \times (1 + g) = \$333.33 \times 1.05 = \$350.00$
$P_2 = P_1 \times (1 + g) = \$350.00 \times 1.05 = \$367.50$
$P_3 = P_2 \times (1 + g) = \$367.50 \times 1.05 = \$385.88$

$r_1 = (DIV_1 + P_1 - P_0) / P_0 = (\$10 + 350.00 - 333.33) / \$333.33 = .08$, or 8%
$r_2 = (DIV_2 + P_2 - P_1) / P_1 = (\$10.50 + 367.50 - 350.00) / \$350.00 = .08$, or 8%
$r_3 = (DIV_3 + P_3 - P_2) / P_2 = (\$11.03 + 385.88 - 367.50) / \$367.50 = .08$, or 8%

Since the rate of return each year is 8 percent, each investor should expect to earn 8%.

Problem [9/100]

True or False: Explain.

a. The value of a share equals the discounted stream of future earnings per share.
b. The value of a share equals the PV of earnings per share assuming the firm does not grow plus the NPV of future growth opportunities.

SOLUTION:

a. False. The value of a share equals the present value of the expected future dividends per share. Earnings per share are not used to calculate share price because a portion of the earnings is used to reinvest in plant, equipment, and working capital.

b. True. The expected return is equal to the yearly dividend divided by the share price. If the firm does not grow and all earnings are paid out as dividends, then the expected return is also equal to the EPS/share price. Therefore, $P_0 = DIV_1 / r = EPS_1 / r$. We must still account for the present value of the growth opportunities, however, so $P_0 = EPS_1 / r + PVGO$.

Problem [10/100]

Free cash flow: Under what conditions does r, a stock's market capitalization rate, equal its earnings-price ratio EPS/P_0?

SOLUTION:

A stock's capitalisation rate equals EPS_1 / P_0 when PVGO = 0, that is when the firm pays out all of its earnings and is not growing.

Problem [13/100]

Horizon value: Suppose the horizon date is set at a time when the firm will run out of positive-NPV investment opportunities. How would you calculate the horizon value? (*Hint*: What is the *P*/*EPS* ratio when PVGO = 0?)

SOLUTION:

If PVGO = 0 at the horizon date, *H*, then:

Horizon value = Earnings forecasted$_{H+1}$ / r

Problem [18/102]

Dividend discount model: Conside the following three stocks:

a. Stock A is expected to provide a dividend of $10 a share forever.
b. Stock B is expected to pay dividend of $5 next year. Thereafter, dividend growth is expected to be 4% a year forever.
c. Stock C is expected to pay a dividend of $5 next year. Thereafter, dividend growth is expected to be 20% a year for five years (i.e., years 2 through 6) and zero thereafter.

If the market capitalisation rate for each stock is 10%, which stock is the most valuable? What if the capitalisation rate is 7%?

SOLUTION:

10 percent capitaliSation rate:

$P_{0\text{ Stock A}} = \text{DIV}_1 / r = \$10 / .1 = \$100$

$P_{0\text{ Stock B}} = \text{DIV}_1 / (r - g) = \$5 / (.1 - .04) = \$83.33$

$$P_{0\text{ Stock C}} = \frac{\text{DIV}_1}{1+r} + \frac{\text{DIV}_1 \times (1+g)}{(1+r)^2} + \frac{\text{DIV}_1 \times (1+g)^2}{(1+r)^3} + \frac{\text{DIV}_1 \times (1+g)^3}{(1+r)^4} + \frac{\text{DIV}_1 \times (1+g)^4}{(1+r)^5} + \frac{\text{DIV}_1 \times (1+g)^5}{(1+r)^6} + \frac{[\text{DIV}_1 \times (1+g)^5 \times (1+g_2)] / r}{(1+r)^6}$$

$P_{0\text{ Stock C}} = \$5 / 1.1 + (\$5 \times 1.2) / 1.1^2 + (\$5 \times 1.2^2) / 1.1^3 + (\$5 \times 1.2^3) / 1.1^4 + (\$5 \times 1.2^4) / 1.1^5 + (\$5 \times 1.2^5) / 1.1^6 + \{[\$5 \times 1.2^5 \times (1 + 0)] / .1\} / 1.1^6$

$P_{0\text{ Stock C}} = \$104.51$

At a 10% capitalisation rate, Stock C has the largest present value.

Using the same formulas as above with a 7% capitalisation rate, the values are:

$P_{0\text{ Stock A}} = \$10 / .07 = \$142.86$

$P_{0 \text{ Stock B}} = \$5 / (.07 - .04) = \$166.67$

$P_{0 \text{ Stock C}} = \$5 / 1.07 + (\$5 \times 1.2) / 1.07^2 + (\$5 \times 1.2^2) / 1.07^3 + (\$5 \times 1.2^3) / 1.07^4 +$
$(\$5 \times 1.2^4) / 1.07^5 + (\$5 \times 1.2^5) / 1.07^6 + \{[\$5 \times 1.2^5 \times (1 + 0)] / .07\} / 1.07^6$

$P_{0 \text{ Stock C}} = \$156.50$

At a 7% capitalisation rate, Stock B has the largest present value.

Problem [20/101]

Two-stage DCF model: Company Q's current return on equity (ROE) is 14%. It pays out one-half of earnings as cash dividends (payout ratio = 0.5). Current book value per share is $50. Book value per share will grow as Q reinvests earnings.

Assume that the ROE and payout ratio stay constant for the next four years. After that, competition forces ROE down to 11.5% and the payout ratio increases to 0.8. The cost of capital is 11.5%

a. What are Q's EPS and dividends next year? How will EPS and dividends grow in years 2, 3, 4, 5, and subsequent years?
b. What is Q's stock worth per share? How does that value depend on the payout ratio and growth rate after year 4?

SOLUTION:

a. Plowback ratio = 1 – payout ratio

Plowback ratio = 1 – .5

Plowback ratio = .5

$g_{Years\ 1\text{-}4}$ = plowback ratio × ROE

$g_{Years\ 1\text{-}4} = .5 \times .14$

$g_{Years\ 1\text{-}4} = .07$

EPS_0 = ROE × book equity per share

$EPS_0 = .14 \times \$50$

$EPS_0 = \$7.00$

DIV_0 = payout ratio × EPS_0

$DIV_0 = .5 \times \$7.00$

$DIV_0 = \$3.50$

$g_{\text{Year 5 and later}}$ = plowback ratio × ROE

$g_{\text{Year 5 and later}} = (1 - .8) \times .115$

$g_{\text{Year 5 and later}} = .023$, or 2.3%

The annual EPS and DIV are as follows:

Year	EPS	DIV
0	$7.00	
1	$\$7.00 \times 1.07 = \$7.49$	$\$7.49 \times .5 = \$3.75$
2	$\$7.00 \times 1.07^2 = \$8.01$	$\$8.01 \times .5 = \$4.01$
3	$\$7.00 \times 1.07^3 = \$8.58$	$\$8.58 \times .5 = \$4.29$
4	$\$7.00 \times 1.07^4 = \$9.18$	$\$9.18 \times .5 = \$4.59$
5	$\$7.00 \times 1.07^4 \times 1.023 = \$9.39$	$\$9.39 \times .8 = \$7.51$

b. $P_H = [\text{DIV}_5 \times (1 + g_2)] / (r - g_2)$

$P_H = (\$7.51 \times 1.023) / (.115 - .023)$

$P_H = \$83.50$

$P_0 = \text{DIV}_1 / (1 + r) + \text{DIV}_2 / (1 + r)^2 + \text{DIV}_3 / (1 + r)^3 + \text{DIV}_4 / (1 + r)^4$
$+ \text{DIV}_5 / (1 + r)^5 + P_H / (1 + r)^5$

$P_0 = \$3.75 / 1.115 + \$4.01 / 1.115^2 + \$4.29 / 1.115^3 + \$4.59 / 1.115^4$
$+ \$7.51 / 1.115^5 + \$83.50 / 1.115^5$

$P_0 = \$65.45$

The last term in the above calculation is dependent on the payout ratio and the growth rate after year 4.

Problem [23/101]

DCF model and PVGO: Financial forecasts for Growth-Tech are given below:

Year	**1**	**2**	**3**	**4**
Book equity	10.00	12.00	14.40	15.55
Earnings per share (EPS)	2.50	3.00	2.30	2.48
Return on Equity (ROE)	0.25	0.25	0.16	0.16
Payout ratio	0.20	0.20	0.50	0.50
Dividends per share (DIV)	0.50	0.60	1.15	1.24
Growth rate of dividends (%)	---	20	92	8

a. Calculate the value of Growth-Tech stock.

b. What part of that value reflects the discounted value of P_3, the price forecasted for year 3?

c. What part of P_3 reflects the present value of growth opportunities (PVGO) after year 3?

d. Suppose that competition will catch up with Growth-Tech by year 4, so that it can earn only its cost of capital on any investments made in year 4 or subsequently. What is Growth-Tech stock worth now under this assumption? (Make additional assumptions if necessary.)

SOLUTION:

a. $P_0 = \text{Div}_1 / (1 + r) + \text{Div}_2 / (1 + r)^2 + \text{Div}_3 / (1 + r)^3 + (\text{Div}_4 / (r - g) / (1 + r)^3$

$P_0 = \$.50 / 1.12 + \$.60 / 1.12^2 + \$1.15 / 1.12^3 + [\$1.24 / (.12 - .08)] / 1.12^3$

$P_0 = \$23.81$

b. The horizon value P_3 contributes:

P_0 = [\$1.24 / (.12 – .08)] / 1.12^3
P_0 = \$22.07

c. Without PVGO, P_3 would equal earnings for year 4 capitalised at 12%, so $PVGO_3$ is valued as:

$PVGO_3 = [DIV_4 / (r - g)] - EPS_4 / r$

$PVGO_3$ = \$1.24 / (.12 – .08) – \$2.48 / .12

$PVGO_3$ = \$10.33

d. The PVGO of \$10.33 is lost at Year 3. Therefore, the current stock price of \$23.81 will decrease by the present value of PVGO:

$P_{0\text{ No-growth}} = P_0 - PVGO_3 / (1 + r)^3$
$P_{0\text{ No-growth}}$ = \$23.81 – \$10.33 / 1.12^3
$P_{0\text{ No-growth}}$ = \$16.45

Problem [27/102]

Valuing free cash flow: Mexican Motors' market cap is 200 billion pesos. Next year's free cash flow is 8.5 billion pesos. Security analysts are forecasting that free cash flow will grow by 7.5% per year for the next five years.

a. Assume that the 7.5% growth rate is expected to continue forever. What rate of return are investors expecting?
b. Mexican Motors has generally earned about 12% on book equity (ROE=12%) and reinvested 50% of earnings. The remaining 50% of earnings has gone to free cash flow. Suppose the company maintains the same ROE and investment rate for the long run. What is the implication for the growth rate of earnings and free cash flow? For the cost of equity?

SOLUTION:

Currency amounts are in millions of pesos.

a. $r = DIV_1 / P_0 + g$
r = 8.5 / 200 + .075
r = .1175, or 11.75%

b. g = ROE x (1-reinvestment rate) = .12 × (0.50)
g = .06, or 6%

$r = DIV_1 / P_0 + g$
r = 8.5 / 200 + .06
r = .1025, or 10.25%

Problem [28/102]

Valuing free cash flow: Phoenix Corp. faltered in the recent recession but is recovering. Free cash flow has grown rapidly. Forecasts made in 2016 are as follows.

$ (millions)	2017	2018	2019	2020	2021
Net income	1.0	2.0	3.2	3.7	4.0
Investment	1.0	1.0	1.2	1.4	1.4
Free cash flow	0.0	1.0	2.0	2.3	2.6

Phoenix's recovery will be complete by 2021, and there will be no further growth in free cash flow.

a. Calculate the PV of free cash flow, assuming a cost of equity of 9%.

b. Assume that Phoenix has 12 million shares outstanding. What is the price per share?

c. If the 2016 net income is $1 million, what is Phoenix's P/E ratio? How do you expect that P/E ratio to change from 2017 to 2021?

d. Confirm that the expected rate of return on Phoenix stock is exactly 9% in each of the years 2017 to 2021.

SOLUTION:

a. $PV_{2016} = DIV_{2017} / (1 + r) + DIV_{2018} / (1 + r)^2 + DIV_{2019} / (1 + r)^3 + DIV_{2020} / (1 + r)^4 + DIV_{2021} / (1 + r)^5 + (DIV_{2021} / r) / (1 + r)^5$

$PV_{2016} = \$0 / 1.09 + \$1 / 1.09^2 + \$2 / 1.09^3 + \$2.3 / 1.09^4 + \$2.6 / 1.09^5 + (\$2.6 / .09) / 1.09^5$

$PV_{2016} = \$24.48$ million

b. Price per $share_{2016} = PV_{2016}$ / number of shares

Price per $share_{2016} = \$24.48 / 12$

Price per $share_{2016} = \$2.04$

c. Based on $1million of net income for 2016:

$P/E_{2016} = \$24.48 / \$1 = 24.48$

The PV of the cash flows at various points in time are as follows:

$PV_{2017} = \$1 / 1.09 + \$2 / 1.09^2 + \$2.3 / 1.09^3 + \$2.6 / 1.09^4 + (\$2.6 / .09) / 1.09^4$

$PV_{2017} = \$26.68$

$PV_{2018} = \$2 / 1.09 + \$2.3 / 1.09^2 + \$2.6 / 1.09^3 + (\$2.6 / .09) / 1.09^3$

$PV_{2018} = \$28.09$

$PV_{2019} = \$2.3 / 1.09 + \$2.6 / 1.09^2 + (\$2.6 / .09) / 1.09^2$

$PV_{2019} = \$28.61$

$PV_{2020} = \$2.6 / 1.09 + (\$2.6 / .09) / 1.09^2$

$PV_{2020} = \$28.89$

$PV_{2021} = \$2.6 + (\$2.6 / .09) / 1.09$

$PV_{2021} = \$28.89$

Thus, the future PE ratios are estimated as:

$PE_{2017} = \$26.68 / \$1 = 26.68$

$PE_{2018} = \$28.09 / \$2 = 14.04$

$PE_{2019} = \$28.61 / \$3.2 = 8.94$

$PE_{2020} = \$28.89 / \$3.7 = 7.81$

$PE_{2021} = \$28.89 / \$4 = 7.22$

d. Using the formula, $r_0 = (DIV_1 + P_1 - P_0) / P_0$, the annual rates of return are:

Rate of $return_{2018}$ = ($1 + 28.09 – 26.68 / $26.68 = .09, or 9%

Rate of $return_{2019}$ = ($2 + 28.61 – 28.09 / $28.09 = .09, or 9%

Rate of $return_{2020}$ = ($2.3 + 28.89 – 28.61) / $28.61 = .09, or 9%

Rate of $return_{2021}$ = ($2.6 + 28.89 – 28.89) / $28.89 = .09, or 9%