FM 2555A – Fall 2016

Solutions to Assignment No. 1
(Questions assigned for submission)
& MARKING SCHEME

♣**Required Assignment Question 1 (Not from the textbook]) [4 points]**

4pts

Mr Williams expects **to retire in 30 years** and would like to accumulate $1 million in his pension fund. If the annual interest rate is 12%, how much should Mr Williams put into his pension fund each month in order to achieve his goal? (Assume that Mr Williams will deposit the same amount each month into his pension fund, using monthly compounding).

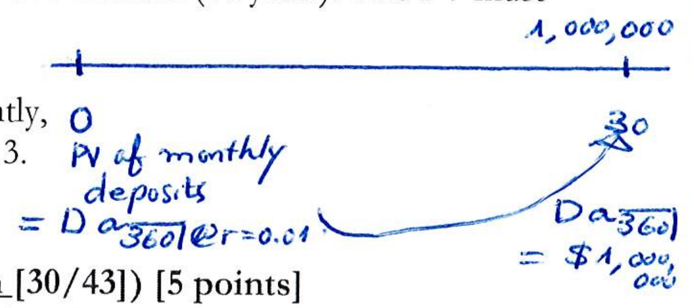
SOLUTION:

1 The monthly rate is $r = 0.01$. There will be $12(30) = 360$ payments until Mr Williams retires. Let D be the required deposit.

1 We take the present value (at time 0) of the monthly deposits, which is an annuity given by $D\,(1 - v^{360})/0.01$, where $v = (1.01)^{-1}$. So, $D\,(1 - v^{360})/0.01 = D(97.21833108)$.

1 Now, take the future value of the annuity at time 360 months (30 years). This FV must equal 1,000, 000. That is,

1 $D(97.21833108)(1.01)^{360} = 1{,}000{,}000$. Consequently, $D = 1{,}000{,}000/[(97.21833108)(1.01)^{360}] = \$286.13$.



♣**Required Assignment Question 2 (Problem [30/43]) [5 points]**

Several years ago *The Wall Street Journal* reported that the winner of the Massachusetts State Lottery prize had the misfortune to be both bankrupt and in prison for fraud. The prize was $9,420,713, to be paid in 19 equal annual installments. (There were 20 installments, but the winner had already received the first payment.) The bankruptcy court judge ruled that the prize should be sold off to the highest bidder and the proceeds used to payy creditors.

a. If the interest rate was 8%, how much would you have been prepared to bid for the prize? *[2.5 pts]*

b. Enhance Reisnurance Company was reported to have offered $4.2 million. Use Excel to find the return that the company was looking for. *[2.5 pts]*

SOLUTION:

2.5 pts

a. $PV = C \times ((1 / r) - \{1 / [r(1 + r)^t]\})$ 1

$PV = (\$9,420,713 / 19) \times ((1 / .08) - \{1 / [.08(1 + .08)^{19}]\})$ 1

$PV = \$4,761,724$ 0.5

b. $PV = C \times ((1 / r) - \{1 / [r(1 + r)^t]\})$

$\$4,200,000 = (\$9,420,713 / 19) \times ((1 / r) - \{1 / [r(1 + r)^t]\})$ 1

Using trial and error in Excel, Newton-Raphson method, or a financial calculator, we find that $r = 9.81\%$. 1.5 [Must be accurate to 2 decimal places.] The method to arrive at the solution must be adequately described.

♣Required Assignment Question 3 (Problem [34/43]) [6 points]

Dear Financial Adviser

My spouse and I are each 62 and hope to retire in three years. After retirement we will receive \$7,500 per month after taxes from our employer's pension plans and \$1,500 per month after taxes from Social Security. Unfortunately our monthly living expenses are \$15,000. Our social obligations preclude further economies.

We have \$1,000,000 invested in a high-grade, free-tax municipal-bond mutual. The return on the fund is 3.5% per year. We plan to make annual withdrawals from the mutual fund to cover the difference between our pension and Social Security income and our living expenses. How many years we run out of money?

Sincerely,
Luxury Challenged
Marblehead, MA

You can assume that the withdrawals (one per year) will sit in a checking account (no interest) until spent. The couple will use the account to cover the monthly shortfalls.

6 pts

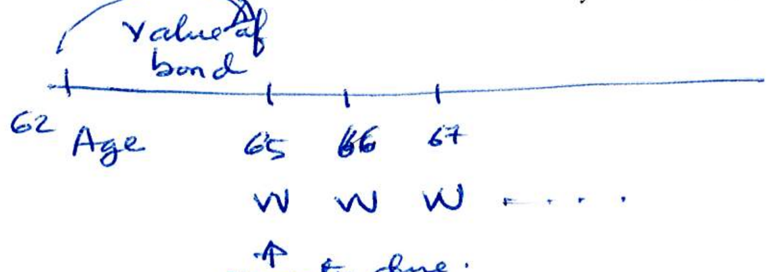
SOLUTION:

$C_t = PV \times (1 + r)^t$

$C_t = \$1,000,000 \times (1.035)^3$

$C_t = \$1,108,718$

↑ at 65



1 Annual retirement shortfall = 12 × (monthly aftertax pension + monthly aftertax Social Security – monthly living expenses)

Annual retirement shortfall = 12 × (\$7,500 + 1,500 – 15,000)

withdrawal = W

1 Annual retirement shortfall = –$72,000

0.5

The withdrawals are an annuity due, so:

1 PV = C × ((1 / r) – {1 / [$r(1 + r)^t$]}) × (1 + r)

$1,108,718 = $72,000 × ((1 / .035) – {1 / [.035(1 + .035)^t]}) × (1 + .035)

14.878127 = (1 / .035) – {1 / [.035(1 + .035)^t]}

1.5 13.693302 = 1 / [.035(1 + .035)^t]

for the calculations .073028 / .035 = 1.035t

t = ln2.086514 / ln1.035

1 t = 21.38 years

♣Required Assignment Question 4 (Problem [33/75]) [5 points]

Duration: The duration of a bond that makes an equal payment each year in perpetuity is (1+yield)/yield. Prove it.

5pts.

SOLUTION:

We begin with the definition of duration as applied to a bond with yield r and an annual payment of C in perpetuity:

$$\mathrm{DUR} = \frac{\dfrac{1C}{1+r}+\dfrac{2C}{(1+r)^2}+\dfrac{3C}{(1+r)^3}+\cdots+\dfrac{tC}{(1+r)^t}+\cdots}{\dfrac{C}{1+r}+\dfrac{C}{(1+r)^2}+\dfrac{C}{(1+r)^3}+\cdots+\dfrac{C}{(1+r)^t}+\cdots}$$

2

We first simplify by dividing both the numerator and the denominator by C:

$$\mathrm{DUR} = \frac{\dfrac{1}{(1+r)}+\dfrac{2}{(1+r)^2}+\dfrac{3}{(1+r)^3}+\cdots+\dfrac{t}{(1+r)^t}+\cdots}{\dfrac{1}{1+r}+\dfrac{1}{(1+r)^2}+\dfrac{1}{(1+r)^3}+\cdots+\dfrac{1}{(1+r)^t}+\cdots}$$

0.5

"$1/r$"

$$\mathrm{DUR} = \frac{PV(C_1)}{PV(Perp)} + \frac{PV(C_2)}{PV(Perp)} + \cdots + \frac{PV(C_t)}{PV(Perp)} + \cdots$$

The denominator is the present value of a perpetuity of \$1 per year, which is equal to $(1/r)$. To simplify the numerator, we first denote the numerator S and then divide S by $(1 + r)$:

$$\frac{S}{(1+r)} = \frac{1}{(1+r)^2} + \frac{2}{(1+r)^3} + \frac{3}{(1+r)^4} + \cdots + \frac{t}{(1+r)^{t+1}} + \cdots \quad 0.5 \quad (*)$$

Note that this new quantity $[S/(1 + r)]$ is equal to the square of denominator in the duration formula above, that is,

If S is the numerator then

$$\frac{S}{(1+r)} = \left(\frac{1}{1+r} + \frac{1}{(1+r)^2} + \frac{1}{(1+r)^3} + \cdots + \frac{1}{(1+r)^t} + \cdots\right)^2 \quad 1 \quad (A)$$

Therefore,

$$\frac{S}{(1+r)} = \left(\frac{1}{r}\right)^2 \Rightarrow S = \frac{1+r}{r^2}$$

Thus, for a perpetual bond paying C dollars per year:

$$\text{DUR} = \frac{1+r}{r^2} \times \frac{1}{(1/r)} = \frac{1+r}{r}$$

1

Let $v = 1/(1+r)$. Right-hand of (A) is

$(v + v^2 + v^3 + \ldots)^2 = (v + v^2 + v^3 + \ldots)(v + v^2 + v^3 + \ldots)$

$= v(v + v^2 + v^3 + \ldots) + v^2(v + v^2 + v^3 + \ldots) + v^3(v + v^2 + v^3 + \ldots) + \ldots$

$= (v^2 + v^3 + v^4 + \ldots) + (v^3 + v^4 + v^5 + \ldots) + (v^4 + v^5 + v^6 + \ldots) + \ldots$

$= v^2 + 2v^3 + 3v^4 + \ldots$

$= \frac{1}{(1+r)^2} + \frac{2}{(1+r)^3} + \frac{3}{(1+r)^4} + \ldots =$ Right-hand side of (*)