FM 2555A
Solutions to Chapter 8 – Practice Exercises

Problem [1/212]
Here are the returns and standard deviations for four investments.

	Return (%)	**Standard Deviation (%)**
Treasury bills	6	0
Stock P	10	14
Stock Q	14.5	28
Stock R	21	26

Calculate the standard deviations of the following portfolios.

a. 50% in Treasury bills, 50% in stock P.
b. 50% each in Q and R, assuming the shares have
   - perfect positive correlation.
   - perfect negative correlation.
   - no correlation.
c. Plot the return-risk profile for Q and R, assuming a correlation coefficient of 0.5
d. Stock Q has a lower return than R but a higher standard deviation. Does that mean that Q's price is too high or that R's price is too low? [Hint: Relate this to the concept of risk measurement.]

SOLUTION:

a. $\sigma = (.5 \times .0) + (.5 \times .14) = .07$, or 7%

b. With perfect positive correlation:
$\sigma = [(.5^2 \times .28^2) + (.5^2 \times .26^2) + 2(.5 \times .5 \times 1 \times .28 \times .26)]^{.5} = .27$, or 27%

With perfect negative correlation:
$\sigma = [(.5^2 \times .28^2) + (.5^2 \times .26^2) + 2(.5 \times .5 \times (-1) \times .28 \times .26)]^{.5} = .01$, or 1%

With no correlation:
$\sigma = [(.5^2 \times .28^2) + (.5^2 \times .26^2) + 2(.5 \times .5 \times 0 \times .28 \times .26)]^{.5} = .191$, or 19.1%

c. See Figure 1 below.

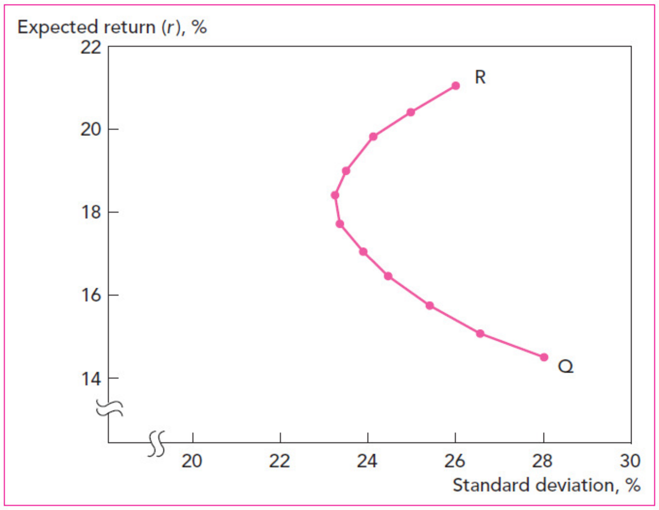


FIGURE 1

d. No, because risk is measured by beta not by standard deviation. Beta measures non-diversifiable risk whilst standard deviation measures total risk. Investors are only compensated with a risk premium for holding non-diversifiable risk.

Problem [2/212]

For each of the following pairs of investments, state which would always be preferred by a rational investor (assuming that these are the *only* investments available to the investor):

a. Portfolio A $r = 18\%$ $\sigma = 20\%$
Portfolio B $r = 14\%$ $\sigma = 20\%$

b. Portfolio C $r = 15\%$ $\sigma = 18\%$
Portfolio D $r = 13\%$ $\sigma = 8\%$

c. Portfolio E $r = 14\%$ $\sigma = 16\%$
Portfolio F $r = 14\%$ $\sigma = 10\%$

SOLUTION:

a. Portfolio A; Investors prefer the higher return given a stated level of risk.

b. Portfolio D; Investors prefer the higher return per unit of risk.

c. Portfolio F; Investors prefer less risk given a stated rate of return.

Problem [3/213]

In Chapter 7, you are given the long-term risk premium of 7.7% and the long-term standard deviation of 19.9% for long-term security returns. Calculate the historical level of the Sharpe ratio of the market portfolio.

SOLUTION:

The long-term risk premium for securities as shown in Chapter 7 is 7.7 percent, and the long-term standard deviation for security returns is 19.9 percent.

Sharpe ratio = 7.7 / 19.9 = .387

Problem [5/213]

a. Plot the following risky portfolios on the graph.

	PORTFOLIO							
Year	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**
Expected return (r), (%)	10	12.5	15	16	17	18	18	20
SD (σ), (%)	23	21	25	29	29	32	35	45

b. Five of these portfolios are efficient, and three are not. Which are inefficient ones?

c. Suppose you can also borrow and lend at an interest rate of 12%. Which of the above portfolios has the highest Sharpe ratio?

d. Suppose you are prepared to tolerate a standard deviation of 25%. What is the maximum expected return that you can achieve if you cannot borrow or lend?

e. What is your optimal strategy if you can borrow or lend at 12% and are prepared to tolerate a standard deviation of 25%? What is the maximum expected return that you can achieve with this risk?

SOLUTION:

a. See Figure 3 below.

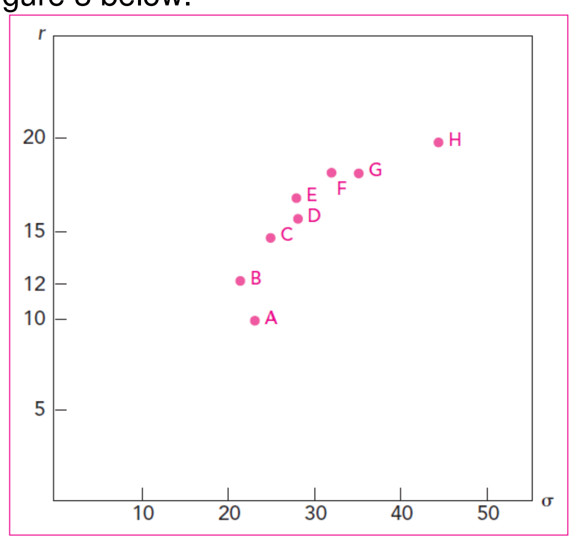


FIGURE 3

b. A, D, G. A is inefficient because an investor could get a higher return with a lower standard deviation (less risk) if he or she invested in Portfolio B instead.

D is inefficient because an investor could earn a slightly higher return at the same risk level if he or she invested in Portfolio E instead. G is inefficient because an investor could earn the same return with less risk if he or she invested in Portfolio F instead.

c. To calculate the Sharpe ratio for each portfolio, subtract the risk-free rate of 12 percent from each portfolio's return and divide that by the standard deviation. The chart below shows the Sharpe ratio for each portfolio. Portfolio F has the highest Sharpe ratio.

(All values are percents)

Portfolio	A	B	C	D	E	F	G	H
Return, r	10	12.5	15	16	17	18	18	20
Standard Deviation, σ	23	21	25	29	29	32	35	45
Sharpe Ratio: $(r - r_f) / \sigma$	-8.7	2.4	12.0	13.8	17.2	**18.8**	17.1	17.8

d. If the maximum standard deviation is 25 percent, then you must find the portfolio with the highest return whose standard deviation is no greater than 25. This return is 15 percent as seen in Portfolio C.

e. Put 25/32 of your money in Portfolio F and lend 7/32 at 12 percent.

$\sigma = 25/32 \times 32\% = 25\%$

$E(r) = 25/32 \times 18\% + 7/32 \times 12\% = 16.7\%$

Problem [6/213]

Suppose that the Treasury bill rate is 6%. Assume that the expected return on the market stays at 9%. You are given the info below.

Stock	Beta
Caterpillar	1.66
Dow Chemical	1.65
Ford	1.44
Microsoft	0.98
Apple	0.91
Johnson & Johnson	0.53
Walmart	0.45
Campbell Soup	0.39
Consolidated Edison	0.17
Newmont	0

a. Calculate the expected return from Johnson & Johnson.

b. Find the highest expected return that is offered by one of these stocks.
c. Find the lowest expected return that is offered by one of these stocks.
d. Would Ford offer a higher or lower expected return if the interest rate were 2% rather than 6%? Assume that the expected market return stays at 9%.
e. Would Walmart offer a higher or lower expected return if the interest rate were 8%?

SOLUTION:

a. $r = .06 + .53(.09 - .06) = .0759$, or 7.59%

b. $r = .06 + 1.66(.09 - .06) = .1098$, or 10.98%

c. $r = .06 + 0(.09 - .06) = .06$, or 6%

d. $r = .06 + 1.44(.09 - .06) = .1032$, or 10.32%
$r = .02 + 1.44(.09 - .02) = .1208$, or 12.08%

If the interest rate were 2%, Ford would offer a higher rate of return.

e. $r = .06 + .45(.09 - .06) = .0735$, or 7.35%
$r = .08 + .45(.09 - .08) = .0845$, or 8.45%

If the interest rate were 8%, Walmart would offer a higher rate of return.

Problem [7/214]

True of False?

a. The CAPM implies that if you could find an investment with a negative beta, its expected return would be less than the interest rate.
b. The expected return on an investment with a beta of 2.0 is twice as high as the expected return on the market.
c. If a stock lies below the security line, it is undervalued.

SOLUTION:

a. Not always true! Whether the expected return is greater than or less than the risk-free rate will depend on the values of the risk-free rate, the market rate, and the negative beta. For example,

$r = .08 + (-.2)(.10 - .08) = .0760$, or 7.60%
$r = .09 + (-.5)(.07 - .09) = .10$, or 10%
$r = .02 + (-.8)(.10 - .02) = -.0440$, or -4.40%

b. False. The risk premium of a security with a beta of 2 will be twice as high as the market risk premium, which is different from the expected return doubling. For example,

$r = .08 + 1(.10 - .08) = .10$, or 10%
$r = .08 + 2(.10 - .08) = .12$, or 12%

c. False. If a stock lies below the security market line it has too low of a return given its level of risk, therefore, it is overvalued.

Problem [8/214]

Consider a three-factor APT model. The factors and associated risk premiums are:

Factor	Risk Premium (%)
Change in GNP	+5
Change in energy prices	-1
Change in long-term interest rates	+2

Calculate expected rates of return on the following stocks. The risk-free interest rate is 7%.

a. A stock whose return is uncorrelated with all three factors.
b. A stock with average exposure to each factor (i.e., with $b = 1$ for each).
c. A pure-play energy stock with high exposure to the energy factor ($b = 2$) but zero exposure to the other two factors.
d. An aluminum company stock with average sensitivity to changes in interest rates and GNP, but negative exposure of $b = -1.5$ to the energy factor. (The aluminum company is energy-intensive and suffers when energy prices rise.)

SOLUTION:

a. $r = .07 + 0(.05) + 0(-.01) + 0(.02) = .07$, or 7%

b. $r = .07 + 1(.05) + 1(-.01) + 1(.02) = .13$, or 13%

c. $r = .07 + 0(.05) + 2(-.01) + 0(.02) = .05$, or 5%

d. $r = .07 + 1(.05) + (-1.5)(-.01) + 1(.02) = .155$, or 15.5%

Problem [9/214]

True or False? Explain or qualify as necessary.

a. Investors demand higher expected rates of return on stocks with more variable rates of return.
b. The CAPM predicts that a security with a beta of 0 will offer a zero expected return.
c. An investor who puts \$10,000 in Treasury bills and \$20,000 in the market portfolio will have a beta of 2.0.
d. Investors demand higher expected rates of return from stocks with returns that are highly exposed to macroeconomic risks.
e. Investors demand higher expected rates of return from stocks with returns that are

very sensitive to fluctuations in the stock market.

SOLUTION:

a. False. Investors demand higher expected rates of return on stocks with more nondiversifiable risk.

b. False. A security with a beta of zero will offer the risk-free rate of return.

c. False. Treasury bills have a beta of 0 and the market has a beta of 1. Therefore, with 1/3 of the investor's money in T-bills and 2/3 of his or her money in the market, the beta will be: $(1/3 \times 0) + (2/3 \times 1) = .67$.

d. True. Macroeconomic risks are nondiversifiable.

e. True. Stock market risks are nondiversifiable.

Problem [11/214]

Mark Harrywitz proposes to invest in two shares, X and Y. He expects a return of 12% from X and 8% from Y. The standard deviation of returns is 8% for X and 5% for Y. The correlation coefficient between the returns is 0.2.

a. Compute the expected return and standard deviation of the followingportfolios:

Portfolio	Percentage in X	Percentage in Y
1	50	50
2	25	75
3	75	25

b. Sketch the set of portfolios composed of X and Y.

c. Suppose that Mr. Harrywitz can also borrow or lend at an interest rate of 5%. Show on your sketch how this alters his opportunities. Given that he can borrow or lend, what proportions of the common stock portfolio should be invested in X and Y?

SOLUTION:

a. The relevant formulas are:

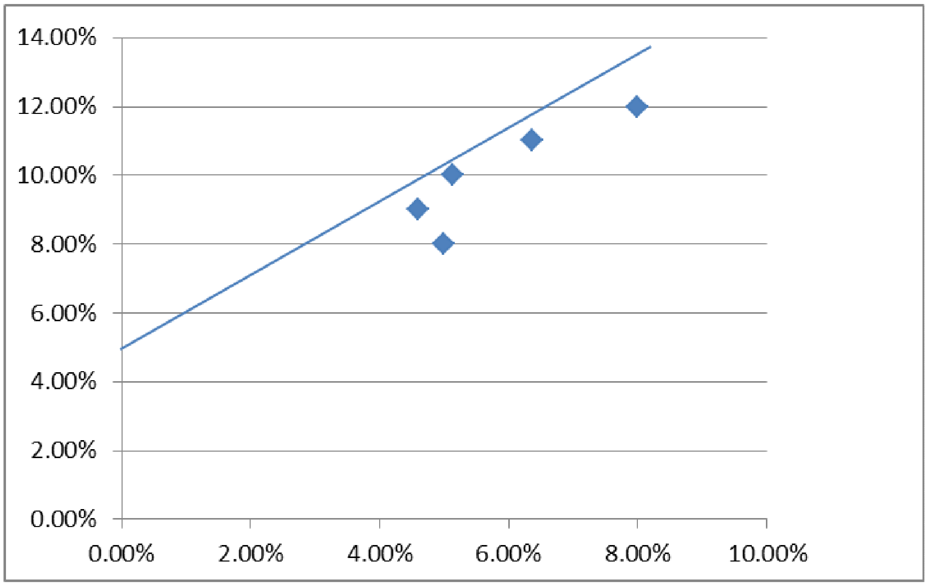
$$r_p = x_1 r_1 + x_2 r_2$$

$$\sigma_p = (x_1^2\sigma_1^2 + 2x_1x_2\sigma_1\sigma_2\rho_{12} + x_2^2\sigma_2^2)^{.5}$$

Assume $r_1 = .12$, $r_2 = .08$, $\sigma_1 = .08$, $\sigma_2 = .05$, and $\rho_{12} = .2$.

Portfolio	x_1	x_2	r_p	σ_p, ρ_{12} = 0.2
X	1.00	.00	12.00	8.00
1	.50	.50	10.00	5.12
2	.25	.75	9.00	4.59
3	.75	.25	11.00	6.37
Y	.00	1.00	8.00	5.00

b. See the figure below. The set of portfolios is represented by the curve indicated by the diamonds.



c. See the figure above. The best opportunities lie along the straight line. From the diagram, the optimal portfolio of risky assets is portfolio 1.

Problem [13/215]

You are given the following data:

	2010	2011	2012	2013	2014
Ms. Sauros	24.90	–.90	18.60	42.10	15.20
S&P 500	17.20	1.00	16.10	33.10	12.70
Interest rate	.12	.04	.06	.02	.02

Calculate the average return and standard deviation of returns for Ms Sauros's portfolio and for the market. Then, calculate the Sharpe ratio for the portfolio and the market. On this measure did Ms Sauros perform better or worse than the market?

SOLUTION: A portfolio's Sharpe ratio is risk premium (excess of average return over risk-free rate) per unit of risk (measured by standard deviation).

	2010	2011	2012	2013	2014	Average Return	Standard Deviation	Sharpe Ratio
Ms. Sauros	24.90	–.90	18.60	42.10	15.20	19.98	13.96	1.0584
S&P 500	17.20	1.00	16.10	33.10	12.70	16.00	10.29	1.0494
Interest rate	.12	.04	.06	.02	.02	5.20		0.00

Ms. Sauros's portfolio performed minimally better than the overall market as seen by her not so significantly greater Sharpe ratio.

Problem [14/215]

Refer to the info in Problem [6/213] above.

a. What is the beta of the portfolio that has 40% invested in Ford and 60% in Johnson & Johnson?
b. Would you invest in this portfolio if you had no superior information about the prospects for these stocks? Devise an alternative portfolio with the same expected return and less risk.
c. Now repeat parts (a) and (b) with a portfolio that has 40% invested in Apple and 60% in Walmart.

SOLUTION:

a. Ford has a beta of 1.44 and Johnson & Johnson's beta is .53. Thus, the portfolio beta is:

$\beta_P = .4 \times 1.44 + .6 \times .53 = .89$

b. Yes, with no superior information about the prospects for these stocks this is the best return for the given level of market risk. However, we can achieve the same return with less unique risk.

We can reduce unique risk by adding more stocks. For example, we could add Microsoft to our portfolio of Ford and Johnson & Johnson and revise the weights to 40 percent, 20 percent, and 40 percent. The beta of the portfolio would be:

$\beta_P = .4 \times .98 + .2 \times 1.44 + .4 \times .53 = .89$

Given the similar beta, this portfolio will have a similar expected return but less unique risk due to the addition of a stock and the diversification benefits of that addition.

c. Apple has a beta of .91 and Walmart's beta is .45. Thus, the portfolio beta is

$\beta_P = .4 \times .91 + .6 \times .45 = 0.63.$

Once again, we can reduce unique risk by adding more stocks. For example, we could add Campbell Soup to our portfolio of Apple and Walmart and revise the weights to 30 percent, 42 percent, and 28 percent. The beta of the portfolio would be:

$\beta_P = .30 \times .39 + .42 \times .91 + .28 \times .45 = .63$

Given the similar beta, this portfolio will have a similar expected return but less unique risk due to the addition of a stock and the diversification benefits of that addition.

Note that several combinations of stocks and weights can be used. The key is to combine three or more securities into a portfolio that has a beta equal to that of the original two-stock portfolio.

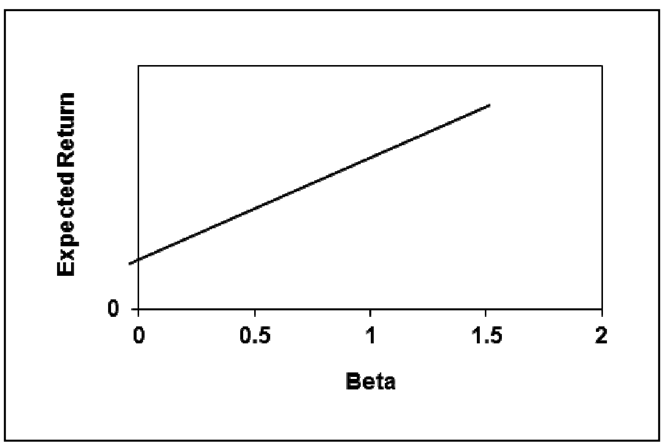
Problem [15/215]

The Treasury bill rate is 4%, and the expected return on the market portfolio is 12%. Using CAPM:

a. Draw a graph showing how the expected return varies with beta.
b. What is the risk premium on the market?
c. What is the required return on investment with a beta of 1.5?
d. If an investment with a beta of 0.8 offers an expected return of 9.8%, does it have a positive NPV?
e. If the market expects a return of 11.2% from stock X, what is beta?

SOLUTION:

a.



b. Market risk premium = $r_m - r_f$
Market risk premium = .12 – .04
Market risk premium = .08, or 8.0%

c. $r = r_f + \beta(r_m - r_f)$

$r = .04 + 1.5(.12 - .04)$
$r = .16$, or 16%

d. For any investment, we can find the opportunity cost of capital using the security market line. With $\beta = .8$, the opportunity cost of capital is:

$r = r_f + \beta(r_m - r_f)$

$r = .04 + .8(.12 - .04)$

$r = .104$, or 10.4%

So, no, not necessarily. The opportunity cost of capital is 10.4 percent and the investment is expected to earn only 9.8 percent. Therefore, the investment has a negative NPV.

e. $r = r_f + \beta(r_m - r_f)$

$.112 = .04 + \beta(.12 - .04)$

$\beta = .9$

Problem [16/216]

Percival Hygiene has $10 million invested in long-term corporate bonds. This bond portfolio's expected annual rate of return is 9%, and the annual standard deviation is 10%.

Amanda Reckonwith, Percival's financial adviser, recommends that Percival consider investing in an index fund that closely tracks the Standard & Poor's 500 Index. The index has an expected return of 14%, and its standard deviation is 16%.

a. Suppose Percival puts all his money in a combination of the index fund and Treasury bills. Can he thereby improve his expected rate of return without changing the risk of his portfolio? The Treasury bill yield is 6%.

b. Could Percival do even better by investing equal amounts in the corporate bond portfolio and the index fund? The correlation between the bond portfolio and the index fund is +0.1.

SOLUTION:

a. Percival's current portfolio provides an expected return of 9% with an annual standard deviation of 10%. First we find the portfolio weights for a combination of Treasury bills (Security 1: standard deviation = 0 percent) and the index fund (Security 2: standard deviation = 16 percent) such that portfolio standard deviation is 10 percent. In general, for a 2-security portfolio:

$\sigma_P^2 = x_1^2\sigma_1^2 + 2x_1x_2\sigma_1\sigma_2\rho_{12} + x_2^2\sigma_2^2$

$.10^2 = 0 + 0 + x_2^2(.16^2)$

$x_2 = .625$

$x_1 = (1 - .625)$

$x_1 = .375$

Given these weights, the portfolio return is:

$r_p = x_1 r_1 + x_2 r_2$

$r_p = .375(.06) + .625(.14)$

$r_p = .110$, or 11.0%

Therefore, he can improve his expected rate of return without changing the risk of his portfolio.

b. With equal amounts in the corporate bond portfolio (Security 1) and the index fund (Security 2), the expected return is:

$r_p = x_1 r_1 + x_2 r_2$

$r_p = .5(.09) + .5(.14)$

$r_p = .115$, or 11.5%

$\sigma_P^2 = x_1^2\sigma_1^2 + 2x_1x_2\sigma_1\sigma_2\rho_{12} + x_2^2\sigma_2^2$

$\sigma_P^2 = .5^2(.10)^2 + 2(.5)(.5)(.10)(.16)(.10) + .5^2(.16)^2$

$\sigma_P^2 = .0097$

$\sigma_P = .0985$, or 9.85%

Therefore, he can do even better by investing equal amounts in the corporate bond portfolio and the index fund. His expected return increases to 11.5% and the standard deviation of his portfolio decreases to 9.85%.