FM 2555A

Solutions to Assignment No. 2
(Only for questions not assigned for submission)

Problem [1/125]

a. What is the payback period on each of the following projects?

Cash Flows ($)					
Project	C_0	C_1	C_2	C_3	C_4
A	-5,000	+1,000	+1,000	+3,000	0
B	-1,000	0	+1,000	+2,000	+3,000
C	-5,000	+1,000	+1,000	+3,000	+5,000

b. Given that you wish to use the payback rule with a cutoff period of two years, which projects would you accept?
c. If you use a cutoff period of three years, which projects would you accept?
d. If the opportunity cost of capital is 10%, which projects have positive NPVs?
e. "If a firm uses a single cutoff period for all projects, it is likely to accept too many short-lived projects." True or false?
f. If the firm uses the discounted-payback rule, will it accept any negative-NPV projects? Explain.

SOLUTION:

a. A = 3 years; B = 2 years; C = 3 years

b. B

c. A, B, and C

d. B and C (At 10%, $NPV_A = –\$1,011$; $NPV_B = \$3,378$; $NPV_C = \$2,405$)

e. True. The payback rule ignores all cash flows after the cutoff date, meaning that future years' cash inflows are not considered. Thus, payback is biased towards short-term projects.

f. It will accept no negative-NPV projects, but will turn down some with positive NPVs. A project can have a positive NPV if all future cash flows are considered but still not meet the stated cutoff period.

Problem [2/125]

Write down the equation defining a project's internal rate of return (IRR). In practice how is IRR calculated?

SOLUTION:

Given the cash flows C_0, C_1, . . . , C_T, IRR is involved in

$$NPV = C_0 + C_1 / (1 + IRR) + C_2 / (1 + IRR)^2 + \ldots + C_T / (1 + IRR)^T = 0$$

IRR is calculated by a numerical method, trial and error and spreadsheet programs (financial calculator).

Problem [3/125]

a. Calculate the net present value of the following project for discount rates of 0, 50, and 100%?

Cash Flows ($)		
C_0	C_1	C_2
-6,750	+4,500	+18,000

b. What is the IRR of the project?

SOLUTION:

a. $NPV = -\$6,750 + \$4,500 / (1 + 0) + \$18,000 / (1 + 0)^2 = \$15,750$
$NPV = -\$6,750 + \$4,500 / (1 + .50) + \$18,000 / (1 + .50)^2 = \$4,250$
$NPV = -\$6,750 + \$4,500 / (1 + 1) + \$18,000 / (1 + 1)^2 = \$0$

b. 100%; NPV = 0 when the discount rate is 100 percent.

Problem [4/125]

You have the chance to participate in a project that produces the following cash flows.

Cash Flows ($)		
C_0	C_1	C_2
+5,000	+4,000	-11,000

The IRR is 13%. If the opportunity cost of capital is 10%, would you accept the offer?

SOLUTION:

No; you would not accept this offer as you are effectively "borrowing" at a rate of interest higher than the opportunity cost of capital. You can verify this decision by proving that the NPV is negative as follows:

$$NPV = \$5,000 + \$4,000 / (1 + .10) + (-\$11,000) / (1 + .10)^2$$
$$NPV = -\$454.55$$

Problem [5/126]

Consider a project with the following cash flows:

Cash Flows ($)		
C_0	C_1	C_2
-100	+200	-75

a. How many IRRs does this project have?
b. Which of the following numbers is the project IRR:
   (i) -50%   (ii) -12%   (iii) +12%   (iv) +50%?
c. The opportunity cost of capital is 20%. Is this an attractive project? Briefly explain.

SOLUTION:

a. Two; because the cash flows change direction twice. Alternatively, you can evaluate the discrimant of the associated quadratic equation.

b. –50% and +50%. The NPV for the project using both of these IRRs is 0.

c. Yes, the NPV is positive at 20 percent

$NPV = –\$100 + \$200 / (1 + .20) + (–\$75) / (1 + .20)^2$
$NPV = \$14.58$

Also, we see in class that NPV is positive to the left of the higher IRR (when there are 2 IRRs).

Problem [7/126]

Suppose you have the following investment opportunities, but only $90,000 available for investment. Which projects should you take?

Project	NPV	Investment
1	5,000	10,000
2	5,000	5,000
3	10,000	90,000
4	15,000	60,000
5	15,000	75,000
6	3,000	15,000

SOLUTION:

1, 2, 4, and 6. The profitability index for each project is shown below:

Project	NPV	Investment	Profitability Index (NPV/Investment)
1	5,000	10,000	5,000 / 10,000 = .5
2	5,000	5,000	5,000 / 5,000 = 1
3	10,000	90,000	10,000 / 90,000 = .11
4	15,000	60,000	15,000 / 60,000 = .25
5	15,000	75,000	15,000 / 75,000 = .2
6	3,000	15,000	3,000 / 15,000 = .2

Start with the project with the highest profitability index and go from there. Project 2 has the highest profitability index and has an initial investment of $5,000. The next highest profitability index is for Project 1, which has an initial investment of $10,000. The next highest is Project 4, which will cost $60,000 up front. So far we have spent $75,000. Projects 5 and 6 both have profitability indexes of

.2, but we only have $15,000 left to spend, so we will add Project 6 to our list. This gives us Projects 1, 2, 4, and 6.

Problem [10/127]

Calculate the IRR(s) for the following project:

C_0	C_1	C_2	C_3
-3,000	+3,500	+4,000	-4,000

For what range of discount rates does the project have positive NPV?

SOLUTION:

	r =	-17.44%	0.00%	10.00%	15.00%	20.00%	25.00%	45.27%
Year 0	–3,000.00	–3,000.00	–3,000.00	–3,000.00	–3,000.00	–3,000.00	–3,000.00	–3,000.00
Year 1	3,500.00	4,239.34	3,500.00	3,181.82	3,043.48	2,916.67	2,800.00	2,409.31
Year 2	4,000.00	5,868.41	4,000.00	3,305.79	3,024.57	2,777.78	2,560.00	1,895.43
Year 3	–4,000.00	-7,108.06	-4,000.00	-3,005.26	-2,630.06	-2,314.81	-2,048.00	-1,304.76
	PV =	-.31	500.00	482.35	437.99	379.64	312.00	-.02

The two IRRs for this project are (approximately): –17.44% and 45.27%. (Alternatively, you can plot the graph of the associated polynomial describing the NPV.)
Between these two discount rates, the NPV is positive.

Problem [2/152]

Mr Art Deco will be paid $100,000 one year hence. This is a nominal flow, which he discounts at an 8% nominal discount rate:

$$PV = 100{,}000/1.08 = \$92{,}593.$$

The inflation rate is 4%. Calculate the PV of Mr Deco's payment using the equivalent *real* cash flow and *real* discount rate. (You should get exactly the same answer as he did.)

SOLUTION:

Real cash flow = $100,000 / (1 + .04) = $96,154

r = (1 + .08) / (1 + .04) – 1 = .03846, or 3.846%

PV = $96,154 / (1 + .03846) = $92,593

Problem [7/153]

Air conditioning for a college dormitory will cost $1.5 million to install and $200,000 per year to operate. The system should last 25 years. The real cost of capital is 5%, and the college pays no taxes. What is the equivalent annual cost?

SOLUTION:

PV of costs = $1,500,000 + $200,000 × ((1 / .05) – {1 / [.05 × (1 + .05)^{25}]})
PV of costs = $4,318,788.91

EAC = \$4,318,788.91 / ((1 / .05) – {1 / [.05 × (1 + .05)^{25}]})
EAC = \$306,428.69

Problem [8/153]

Machines A and B are mutually exclusive and are expected to produce the following real cash flows:

Cash Flows ($ thousands)				
Machine	C_0	C_1	C_2	C_3
A	-100	+110	+121	
B	-120	+110	+121	+133

The real opportunity cost of capital is 10%.

a. Calculate the NPV of each machine.
b. Calculate the equivalent annual cash flow from each machine.
c. Which machine should you buy?

SOLUTION:

a. NPV_A = –\$100,000 + \$110,000 / (1 + .10) + \$121,000 / (1 + .10)^2
NPV_A = \$100,000

NPV_B = –\$120,000 + \$110,000 / (1 + .10) + \$121,000 / (1 + .10)^2 + \$133,000 / (1 + .10)^3
NPV_B = \$179,925

b. $EACF_A$ = \$100,000 / ((1 / .10) – {1 / [.10(1 + .10)^2]})
$EACF_A$ = \$57,619

$EACF_B$ = \$179,925 / ((1 / .10) – {1 / [.10(1 + .10)^3]})
$EACF_B$ = \$72,350

c. Select Machine B because it has the higher equivalent annual cash flow.

Problem [9/153]

Machine C was purchased five years ago for \$200,000 and produces an annual real cash flow of \$80,000. It has no salvage value but is expected to last another five years. The company can replace machine C with machine B (see Problem 8/153) *either* now *or* at the end of five years. What should it do?

SOLUTION:

NPV_B = –\$120,000 + \$110,000 / (1 + .10) + \$121,000 / (1 + .10)^2 + \$133,000 / (1 + .10)^3
NPV_B = \$179,925

$EACF_B$ = \$179,925 / ((1 / .10) – {1 / [.10(1 + .10)^3]})
$EACF_B$ = \$72,350

In this problem, we must ignore the sunk costs and past real cash flows and focus on future cash flows.

Machine C is expected to last another five years and produces a real annual cash flow of $80,000.

Since Machine C's real annual cash flow exceeds Machine B's equivalent annual cash flow, the company should wait and replace Machine C at the end of five years.

Problem [11/153]

CSC is evaluating a new project to produce encapsulators. The initial investment in plant and equipment is $500,000. Sales of encapsulators in year 1 are forecasted at $200,000 and costs at $100,000. Both are expected to increase by 10% a year in line with inflation. Profits are taxed at 35%. Working capital in each year consists of inventories of raw materials and is forecasted at 20% of sales in the following year.

The project will last five years and the equipment can be depreciated straight-line over these five years. If the nominal discount rate is 15%, show that the net present value of the project is the same whether calculated using real cash flows or nominal cash flows.

SOLUTION:

Nominal rate = 15%

Inflation rate = 10%

$R_{real} = [(1 + .15) / (1 + .10)] – 1$

$R_{real} = .045455$, or 4.5455%

(figures in $) Year:	0	1	2	3	4	5
Revenues		200,000	220,000	242,000	266,200	292,820
Costs		100,000	110,000	121,000	133,100	146,410
Depreciation		100,000	100,000	100,000	100,000	100,000
Pretax profit		0	10,000	21,000	33,100	46,410
Taxes at 35%		0	3,500	7,350	11,585	16,244
Aftertax profit		0	6,500	13,650	21,515	30,167
Working capital	40,000	44,000	48,400	53,240	58,564	0
Operating cash flow		100,000	106,500	113,650	121,515	130,167
Change in working capital	-40,000	-4,000	-4,400	-4,840	-5,324	58,564

Capital investment	-500,000	0	0	0	0	0
Net cash flows (nominal)	–540,000	96,000	102,100	108,810	116,191	188,731
NPV (nominal) at 15%	**–147,510**					
Net cash flows (real) (10% inflation)*	–540,000	87,273	84,380	81,751	79,360	117,187
NPV (real) at 4.5455%	**–147,510**					

* Real cash flow$_t$ = Nominal cash flow$_t$ / (1 + inflation rate)^t

Problem [14/154]

Ms T Potts, the treasurer of Ideal China, has a problem. The company has just ordered a new kiln for $400,000. Of this sum, $50,000 is described by the supplier as an installation cost. Ms Potts does not know whether the Internal Revenue Service (IRS) will permit the company to treat this cost as a tax-deductible current expense or as a capital investment. In the latter case, the company could depreciate the $50,000 using the five-year MACRS tax depreciation schedule. How will the IRS's decision affect the after-tax cost of the kiln? The tax rate is 35% and the opportunity cost of capital is 5%.

SOLUTION:

If the $50,000 installation cost is expensed at the end of year 1, the value of the tax shield is

$$PV = (\$50,000 \times .35) / (1 + .05)$$

$$PV = \$16,667$$

If the $50,000 cost is capitalised and then depreciated using a five-year MACRS depreciation schedule, the value of the tax shield is

$$PV = (.35 \times \$50,000) \times (.20 / (1 + .05) + .32 / (1 + .05)^2 + .192 / (1+.05)^3 + .1152 / (1 + .05)^4 + .1152 / (1 + .05)^5 + .0576 / (1 + .05)^6)$$

$$PV = \$15,306$$

If the installation cost can be expensed, then the tax shield is larger, which means the after-tax cost is smaller.

Problem [16/154]

A project requires an initial investment of $100,000 and is expected to produce a cash inflow before tax of $26,000 per year for five years. Company A has substantial accumulated tax losses and is unlikely to pay taxes in the foreseeable future. Company B pays corporate taxes at a rate of 35% and can depreciate the investment for tax purposes using the five-year MACRS tax depreciation schedule. Suppose the opportunity cost of capital is 8%. Ignore inflation.

a. Calculate the project NPV for each company.
b. What is the IRR of the after-tax cash flows for each company? What does comparison of the comparison of the IRRs suggest is the effective corporate tax rate?

SOLUTION:

a. $NPV_A = –\$100,000 + \$26,000 \times ((1 / .08) – \{1 / [.08(1 + .08)^5]\})$

$NPV_A = \$3,810$

NPV_B = –investment + PV(after-tax cash flow) + PV(depreciation tax shield)

$NPV_B = –\$100,000 + [\$26,000 \times (1 – .35)] \times ((1 / .08) – \{1 / [.08 \times (1 + .08)^5]\})$

$+ (.35 \times \$100,000) \times [.20 / (1 + .08) + .32 / (1 + .08)^2 + .192 / (1 + .08)^3$

$+ .1152 / (1 + .08)^4 + .1152 / (1 + .08)^5 + .0576 / (1 + .08)^6]$

$NPV_B = –\$4,127$

b. To calculate the effective tax rate, first compute the project cash flows for each year. For years 1 and after, you can use this formula:

After-tax cash $flow_t$= (pretax cash $flow_t$ × (1 – tax rate) + (initial investment × depreciation $rate_t$ × tax rate)

After-tax cash flows:

Year:	0	1	2	3	4	5	6
Company A	–100,000	26,000	26,000	26,000	26,000	26,000	0
Company B	–100,000	23,900	28,100	23,620	20,932	20,932	2,016

$IRR_A = 9.43\%$
$IRR_B = 6.39\%$

Effective tax rate = 1 – (.0639 / .0943) = .323, or 32.3%

Problem [1/186]

A game of chance offers the following odds and payoffs. Each play of the game costs $100, so the net profit per play is the payoff less $100.

Probability	***Payoff***	***Net Profit***
0.10	$500	$400
0.50	100	0
0.40	0	-100

What are the expected cash payoff and expected rate of return? Calculate the variance and standard deviation of this rate of return

SOLUTION:

Expected payoff = (.10 × \$500) + (.50 × \$100) + (.40 × \$0)
= \$100

Rates of return:
(\$500 – 100) / \$100 = 400%
(\$100 – 100) / \$100 = 0%
(\$0 – 100) / \$100 = –100%

Expected rate of return = (.10 × 400%) + (.50 × 0%) + (.40 × –100%)
Expected rate of return = 0%

Variance = $.10(400\% - 0)^2 + .50(0\% - 0)^2 + .40(-100\% - 0)^2$
Variance = 20,000

Standard deviation = $20{,}000^{.5}$
Standard deviation = 141.42%

Problem [2/186]

The following table shows the nominal returns on the US stocks and the rate of inflation.

a. What was the standard deviation of the nominal returns?
b. Calculate the arithmetic average real return.

Year	*Nominal Return (%)*	*Inflation (%)*
2010	17.2	1.5
2011	1.0	3.0
2012	16.1	1.7
2013	33.1	1.5
2014	12.7	0.8

SOLUTION:

a. Average nominal return = (.172 + .010 + .161 + .331 + .127) / 5
Average nominal return = .1602, or 16.02%

Variance = $[(.172 - .1602)^2 + (.010 - .1602)^2 + (.161 - .1602)^2 + (.331 - .1602)^2 + (.127 - .1602)^2] / 5$
Variance = .010595

Standard deviation = $.010595^{.5}$
Standard deviation = .1029, or 10.29%

b. Average real return = {[(1.172 / 1.015) – 1] + [(1.010 / 1.030) – 1] + [(1.161 / 1.017) – 1] + [(1.331 / 1.015) – 1] + [(1.127 / 1.008) – 1]} / 5
Average real return = .1412, or 14.12%

Problem [3/187]

During the boom years of 2010-2014, ace mutual fund manager Diana Sauros produced the following percentage rates of return. Rates of return on the market are given for comparison.

	C_0	C_1	C_2	C_3	C_4
Ms Sauros	+24.9	-0.9	+18.6	+42.1	+15.2
S&P 500	+17.2	+1.0	+16.1	+33.1	+12.7

Calculate the average return and standard deviation of Ms Sauro's mutual fund. Did she do better or worse than the market by these measures?

SOLUTION:

Ms. Sauros:

Average return = [.249 + (–.009) + .186 + .421 + .152] / 5

Average return = .1998, or 19.98%

Variance = $[(.249 - .1998)^2 + (-.009 - .1998)^2 + (.186 - .1998)^2 + (.421 - .1998)^2 + (.152 - .1998)^2] / 5$

Variance = .019485

Standard deviation = .0194

Standard deviation = .1396, or 13.96%

S&P 500:

Average return = (.172 + .010 + .161 + .331 +.127) / 5

Average return = .1602, or 16.02%

Variance = $[(.172 - .1602)^2 + (.010 - .1602)^2 + (.161 - .1602)^2 + (.331 - .1602)^2 + (.127 - .1602)^2$

Variance = .010595

Standard deviation = $.010595^{.5}$

Standard deviation = .1029, or 10.29%

Problem [6/187]

To calculate the variance of a three-stock portfolio, you need to add nine boxes:

Use the same symbols that we used in the class; for example, x_1 = proportion invested in stock 1 and σ_{12} = covariance between stocks 1 and 2. Now complete the nine boxes.

SOLUTION:

$x_1^2\sigma_1^2$	$x_1x_2\sigma_{12}$	$x_1x_3\sigma_{13}$
$x_1x_2\sigma_{12}$	$x_2^2\sigma_2^2$	$x_2x_3\sigma_{23}$
$x_1x_3\sigma_{13}$	$x_2x_3\sigma_{23}$	$x_3^2\sigma_3^2$

Problem [8/188]
A portfolio contains equal investments in 10 stocks. Five have a beta of 1.2; the remainder have a beta of 1.4. What is the portfolio beta?
a. 1.3
b. Greater than 1.3 because the portfolio is not completely diversified.
c. Less than 1.3 because diversification reduces beta.
SOLUTION:
$\beta_p = \{(5 \times 1.2) + [(10 - 5) \times 1.4)]\} / 10$
$\beta_p = 1.3$

Beta measures systematic risk which cannot be eliminated by diversification.

Problem [10/188]
Here are the inflation rates and US stock market and Treasury bill returns between 1929 and 1933:

Year	Inflation	Stock Market Return	T-Bill Return
1929	-0.2	-14.5	4.8
1930	-6.0	-28.3	2.4
1931	-9.5	-43.9	1.1
1932	-10.3	-9.9	1.0
1933	0.5	57.3	0.3

a. What was the real return on the stock market in each year?
b. What was the arithmetic average real return?
c. What was the risk premium in each year?
d. What was the average risk premium?
e. What was the standard deviation of the risk premium?
SOLUTION:
a. $r = [(1 + R) / (1 + i)] - 1$

$r_{1929} = \{[1 + (-.145)] / (1 + .002)\} - 1 = -.1467$, or -14.67%
$r_{1930} = \{[1 + (-.283)] / [1 + (-.060)]\} - 1 = -.2372$, or -23.72%
$r_{1931} = \{[1 + (-.439)] / [1 + (-.095)]\} - 1 = -.3801$, or -38.01%
$r_{1932} = \{[1 + (-.099)] / [1 + (-.103)]\} - 1 = .0045$ or $.45\%$

r_{1933} = [(1 + .573) / (1 + .005)] – 1 = .5652, or 56.52%

b. Average real return = [–.1467 + (–.2372) + (–.3801) + .0045 + .5652] / 5
Average real return = –.0382, or –3.82%

c. Risk $premium_{1929}$ = –.145 – .048 = –.1930, or –19.30%
Risk $premium_{1930}$ = –.283 – .024 = –.3070, or –30.70%
Risk $premium_{1931}$ = –.439 – .011 = –.4500, or –45.00%
Risk $premium_{1932}$ = –.099 – .010 = –.1090, or –10.90%
Risk $premium_{1933}$ = .573 – .003 = .5700, or 57.00%

d. Average risk premium = [–.1930 + (–.3070) + (–.4500) + (–.1090) + .5700] /5
Average risk premium = –0978, or –9.78%

e. $\sigma_{\text{Risk premium}} = \{[-.1930 - (-.0978)]^2 + [-.3070 - (-.0978)]^2 + [-.4500 - (-.0978)]^2$

$+ [-.1090 - (-.0978)]^2 + [.5700 - (-.0978)]^2]\} / 5$
$\sigma_{\text{Risk premium}}$ = .1246, or 12.46%

Problem [13/188]

Lonesome Gulch Mines has a standard deviation of 42% per year and a beta of +0.10. Amalgamated Cooper has a standard deviation of 31% a year and a beta of +0.66. Explain why Lonesome Gulch is the safer investment for a diversified investor.

SOLUTION:

In the context of a well-diversified portfolio, the only risk characteristic of a single security that matters is the security's contribution to the overall portfolio risk. This contribution is measured by beta. Lonesome Gulch is the safer investment for a diversified investor because its beta of .10 is lower than the beta of Amalgamated Copper of .66. For a diversified investor, the standard deviations are irrelevant.

Problem [19/189]

There are few, if any, real companies with negative betas. But suppose you found one with beta = -0.25.

a. How would you expect this stock's rate of return to change if the overall market rose by an extra 5%? What if the market fell by an extra 5%?
b. You have $1 million in a well-diversified portfolio of stocks. Now you receive an additional $20,000 bequest. Which of the following actions will yield the safest overall portfolio return?
   i. Invest $20,000 in Treasury bills (which have beta = 0).
   ii. Invest $20,000 in stocks with beta = 1.
   iii. Invest $20,000 in the stocks with beta = -0.25.

SOLUTION:

a-1. Change in stock's rate of return = .05 × –.25 = –.0125, or –1.25%

a-2. Change in stock's rate of return = –.05 × –.25 = .0125, or 1.25%

b. "Safest" implies lowest risk. Assuming the well-diversified portfolio is invested in typical securities, the portfolio beta is approximately one. The largest reduction in beta is achieved by investing the $20,000 in a stock with the negative beta.