HOW TO CALCULATE PRESENT VALUES

Topics Covered

- Future Values and Present Values
- Looking for Shortcuts—Perpetuities and Annuities
- More Shortcuts—Growing Perpetuities and Annuities
- How Interest Is Paid and Quoted

Present Value and Future Value

Present Value

Value today of a future cash flow.

Future Value

Amount to which an investment will grow after earning interest

Future Values

Future Value of $100 = FV

$$\boxed{FV = \$100 \times (1 + r)^t}$$

Future Values

$$\boxed{\text{FV} = \$100 \times (1+r)^t}$$

$$$$$$$

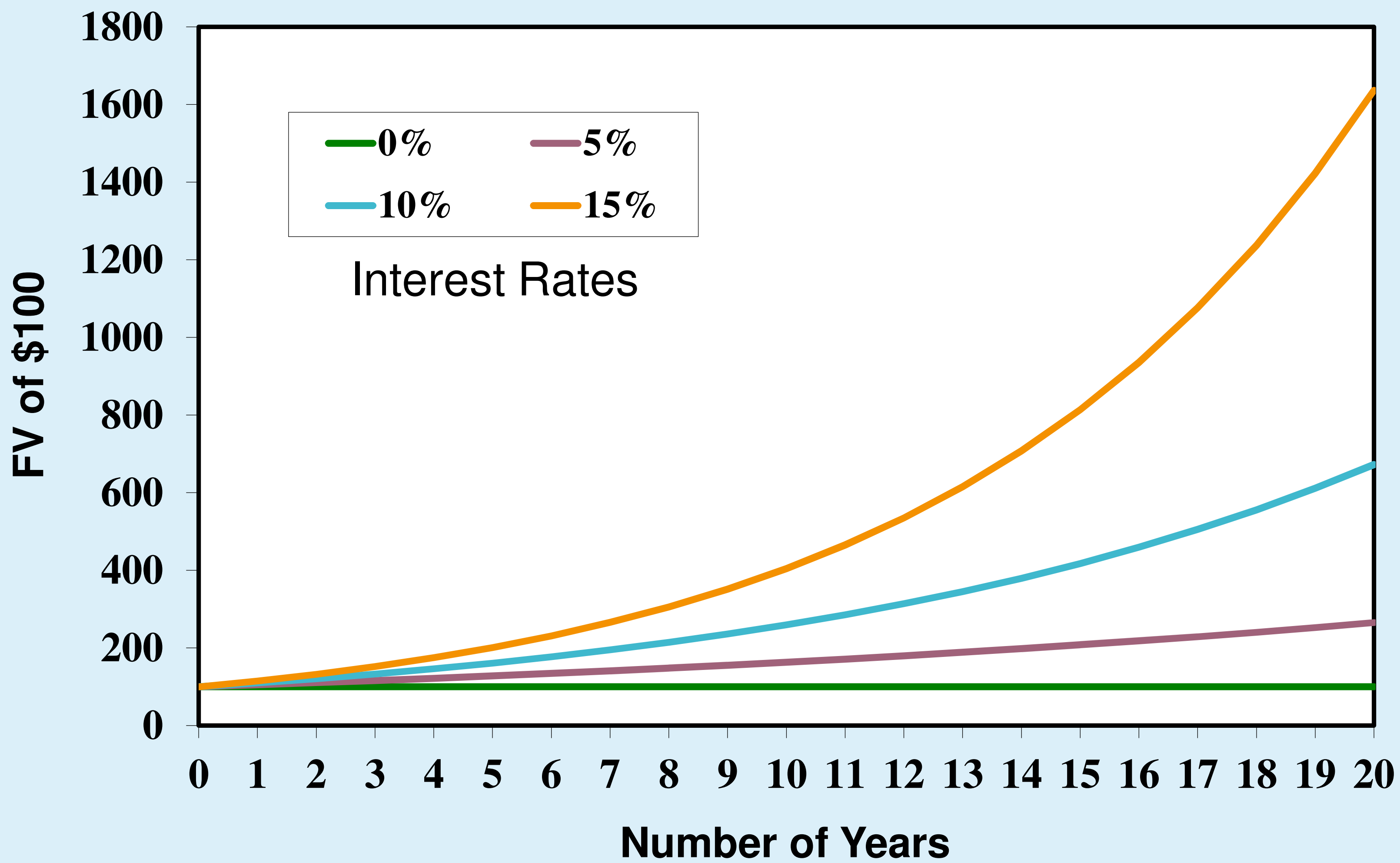
Example - FV

What is the future value of $100 if interest is compounded annually at a rate of 7% for two years?

$$FV = \$100 \times (1.07) \times (1.07) = 114.49$$

$$FV = \$100 \times (1+.07)^2 = \$114.49$$

Future Values with Compounding



Present Value

Present value = PV

$$\boxed{\text{PV} = \text{discount factor} \times C_1}$$

Present Value

Discount factor = DF = PV of $1

$$\boxed{DF = \frac{1}{(1+r)^t}}$$

Discount factors can be used to compute the present value of any cash flow

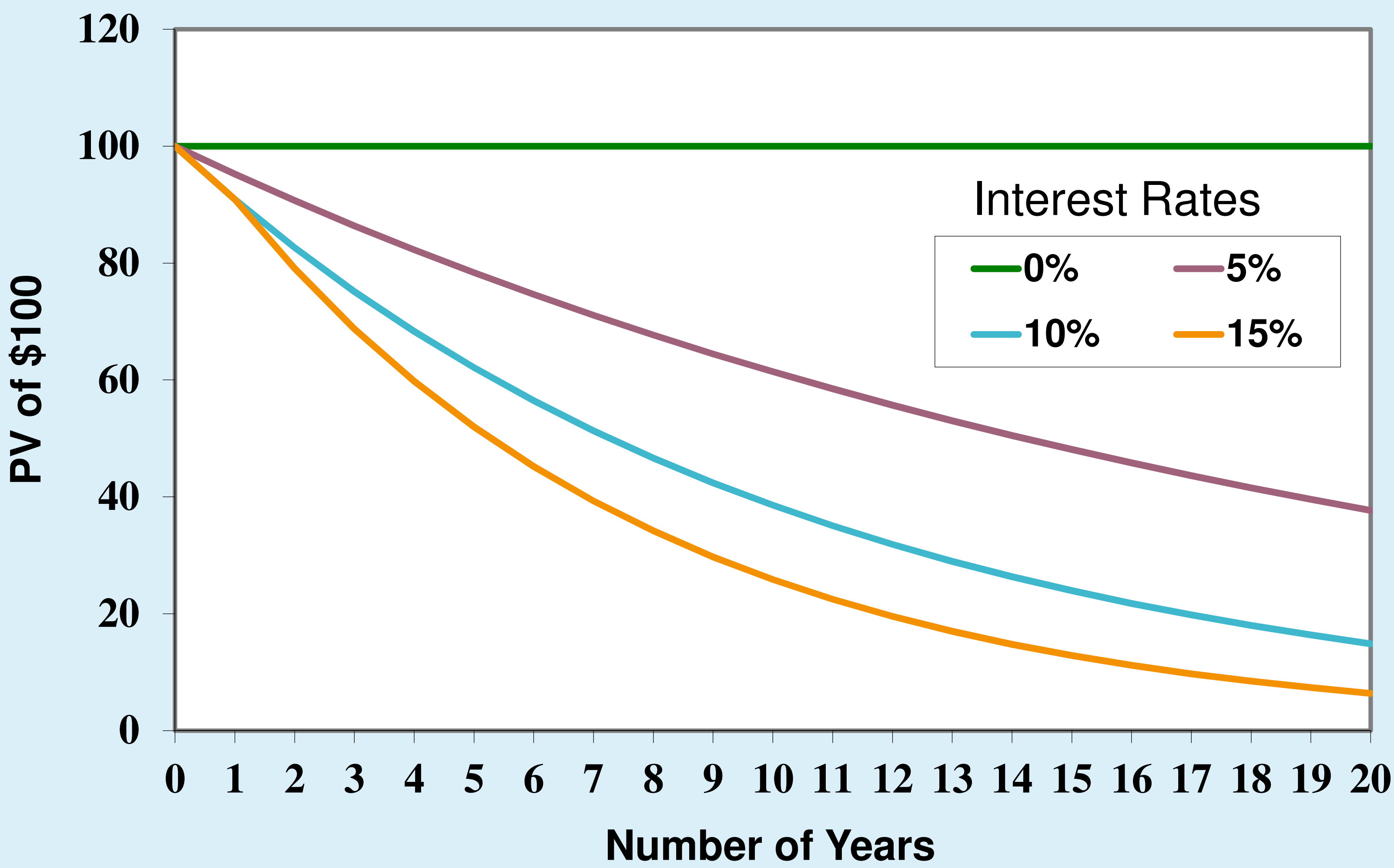
Present Value

- The PV formula has many applications. Given any variables in the equation, you can solve for the remaining variable. Also, you can reverse the prior example.

$$\mathrm{PV} = \mathrm{DF}_2 \times C_2$$

$$\mathrm{PV} = \frac{1}{(1+.07)^2} \times 114.49 = 100$$

Present Values with Compounding



Valuing an Office Building

Step 1: Forecast cash flows

Cost of building = C_0 = 700,000

Sale price in Year 1 = C_1 = 800,000

Step 2: Estimate opportunity cost of capital

If equally risky investments in the capital market offer a return of 7%, then

Cost of capital = r = 7%

Valuing an Office Building

Step 3: Discount future cash flows

$$\text{PV} = \frac{C_1}{(1+r)} = \frac{800{,}000}{(1+.07)} = 747{,}664$$

Step 4: Go ahead if PV of payoff exceeds investment

$$\begin{aligned}\text{NPV} &= 747{,}664 - 700{,}000 \\ &= 47{,}664\end{aligned}$$

Net Present Value

$$\text{NPV} = \text{PV - required investment}$$

$$\text{NPV} = C_0 + \frac{C_1}{1+r}$$

Risk and Present Value

- Higher risk projects require a higher rate of return
- Higher required rates of return cause lower PVs

$$\text{PV of } C_1 = \$800{,}000 \text{ at } 7\%$$

$$\text{PV} = \frac{800{,}000}{1+.07} = 747{,}664$$

Risk and Present Value

$$\text{PV of } C_1 = \$800{,}000 \text{ at } 12\%$$

$$\text{PV} = \frac{800{,}000}{1+.12} = 714{,}286$$

$$\text{PV of } C_1 = \$800{,}000 \text{ at } 7\%$$

$$\text{PV} = \frac{800{,}000}{1+.07} = 747{,}664$$

Risk and Net Present Value

$$NPV = PV - \text{required investment}$$

$$NPV = 714{,}286 - 700{,}000$$
$$= \$14{,}286$$

Net Present Value Rule

- Accept investments that have positive net present value

<u>Example</u>

Use the original example. Should we accept the project given a 10% expected return?

$$\text{NPV} = -700{,}000 + \frac{800{,}000}{1.10} = \$27{,}273$$

Rate of Return Rule

- Accept investments that offer rates of return in excess of their opportunity cost of capital

Example

In the project listed below, the foregone investment opportunity is 12%. Should we do the project?

$$\text{Return} = \frac{\text{profit}}{\text{investment}} = \frac{800{,}000 - 700{,}000}{700{,}000} = .143 \text{ or } 14.3\%$$

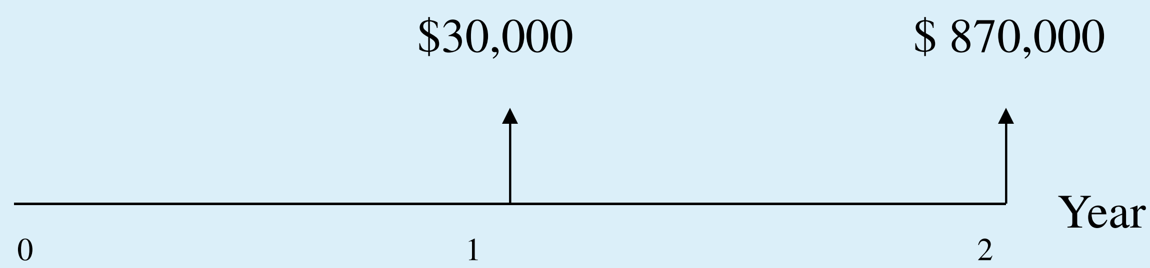
Multiple Cash Flows

For multiple periods we have the discounted cash flow (DCF) formula

$$\boxed{PV_0 = \frac{C_1}{(1+r)^1} + \frac{C_2}{(1+r)^2} + \dots + \frac{C_t}{(1+r)^t}}$$

$$\boxed{NPV_0 = C_0 + \sum_{t=1}^{T} \frac{C_t}{(1+r)^t}}$$

Net Present Values



Present value

Year 0

-$700,000

$30,000/1.12 = \$26,786$

$870,000/1.12^2 = \underline{\$693,559}$

Total = $20,344

Shortcuts

- Sometimes there are shortcuts that make it very easy to calculate the present value of an asset that pays off in different periods. These tools allow us to cut through the calculations quickly.

Shortcuts

Perpetuity - Financial concept in which a cash flow is theoretically received forever.

$$\text{Return} = \frac{\text{cash flow}}{\text{present value}}$$

$$r = \frac{C}{\text{PV}}$$

Shortcuts

Perpetuity - Financial concept in which a cash flow is theoretically received forever.

$$\text{PV of cash flow} = \frac{\text{cash flow}}{\text{discount rate}}$$

$$\text{PV}_0 = \frac{C_1}{r}$$

Present Values

Example

What is the present value of $1 billion every year, for all eternity, if you estimate the perpetual discount rate to be 10%?

$$PV = \frac{\$1\,\text{bil}}{0.10} = \$10\ \text{billion}$$

Present Values

Example - continued

What if the investment does not start making money for 3 years?

$$PV = \frac{\$1 \text{ bil}}{0.10} \times \left(\frac{1}{1.10^{3}}\right) = \$7.51 \text{ billion}$$

How to Value Annuities

Annuity - An asset that pays a fixed sum each year for a specified number of years

$$\text{PV of annuity} = C \times \left[\frac{1}{r} - \frac{1}{r(1+r)^t} \right]$$

Perpetuities & Annuities

<u>PV Annuity Factor (PVAF)</u> - The present value of $1 a year for each of *t* years

$$\text{PVAF} = \left[\frac{1}{r} - \frac{1}{r(1+r)^t}\right]$$

Short Cuts

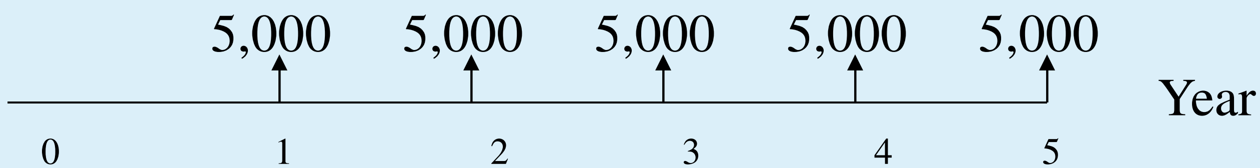
Annuity - An asset that pays a fixed sum each year for a specified number of years.

Asset	Year of Payment 1 2.....t t + 1	Present Value
Perpetuity (first payment in year 1)	→	$\dfrac{C}{r}$
Perpetuity (first payment in year t + 1)	→	$\left(\dfrac{C}{r}\right)\dfrac{1}{(1+r)^t}$
Annuity from year 1 to year t	→	$\left(\dfrac{C}{r}\right)-\left(\dfrac{C}{r}\right)\left(\dfrac{1}{(1+r)^t}\right)$

Costing an Installment Plan

Example

Tiburon Autos offers you "easy payments" of $5,000 per year, at the end of each year for 5 years. If interest rates are 7%, per year, what is the cost of the car?



Present Value
at year 0

$$5{,}000/1.07 = 4{,}673$$

$$5{,}000/(1.07)^2 = 4{,}367$$

$$5{,}000/(1.07)^3 = 4{,}081$$

$$5{,}000/(1.07)^4 = 3{,}814$$

$$5{,}000/(1.07)^5 = 3{,}565$$

$$\text{Total NPV} = 20{,}501$$

Winning Big at the Lottery

Example

The state lottery advertises a jackpot prize of $590.5 million, paid in 30 installments over 30 years of $19.683 million per year, at the end of each year. If interest rates are 3.6% what is the true value of the lottery prize?

$$\text{Lottery value} = 19.683 \times \left[\frac{1}{.036} - \frac{1}{.036(1+.036)^{30}} \right]$$

$$\text{Value} = \$357.5 \text{ million}$$

Annuity Due

Annuity due - Level stream of cash flows starting immediately

How does it differ from an ordinary annuity?

$$\boxed{\mathrm{PV}_{\text{Annuity due}} = \mathrm{PV}_{\text{Annuity}} \times (1 + r)}$$

How does the future value differ from an ordinary annuity?

$$\boxed{\mathrm{FV}_{\text{Annuity due}} = \mathrm{FV}_{\text{Annuity}} \times (1 + r)}$$

Annuities Due: Example

$$\mathrm{FV}_{\mathrm{AD}} = \mathrm{FV}_{\mathrm{Annuity}} \times (1 + r)$$

<u>Example</u>: Suppose you invest $429.59 annually at the beginning of each year at 10% interest. After 50 years, how much would your investment be worth?

$$FV_{AD} = \$429.59 \times \left[\frac{1}{.10} - \frac{1}{.10(1+.10)^{50}}\right] \times 1.10^{50} \times 1.10$$
$$= \$550{,}003.81$$

Paying Off a Bank Loan

Example - Annuity

You are purchasing a TV for $1,000. You are scheduled to make 4 annual installments. Given a rate of interest of 10%, what is the annual payment?

$$\$1{,}000 = \text{PMT} \times \left[\frac{1}{.10} - \frac{1}{.10(1+.10)^4} \right]$$

$$\text{PMT} = \$315.47$$

FV Annuity Short Cut

Future Value of an Annuity – The future value of an asset that pays a fixed sum each year for a specified number of years.

$$\text{FV of annuity} = C \times \left[\frac{(1+r)^t - 1}{r} \right]$$

FV Annuity Short Cut

Example

What is the future value of $20,000 paid at the end of each of the following 5 years, assuming your investment returns 8% per year?

$$\mathrm{FV} = 20{,}000 \times \left[\frac{(1+.08)^5 - 1}{.08} \right]$$

$$= \$117{,}332$$

Constant Growth Perpetuity

$$\boxed{\mathrm{PV}_0 = \frac{C_1}{r - g}}$$

g = the annual growth rate of the cash flow

Constant Growth Perpetuity

NOTE: This formula can be used to value a perpetuity at any point in time.

$$\mathrm{PV}_0 = \frac{C_1}{r - g}$$

$$\boxed{PV_t = \frac{C_{t+1}}{r - g}}$$

Constant Growth Perpetuity

Example

What is the present value of $1 billion paid at the end of every year in perpetuity, assuming a rate of return of 10% and a constant growth rate of 4%?

$$PV_0 = \frac{1}{.10 - .04}$$

$$= \$16.667 \text{ billion}$$

Effective Interest Rates

Effective Annual Interest Rate - Interest rate that is annualized using compound interest

Annual Percentage Rate - Interest rate that is annualized using simple interest

EAR & APR Calculations

Annual Percentage Rate (APR):

$$\mathrm{APR} = \mathrm{MR} \times 12$$

Effective Annual Interest Rate (EAR):

$$\mathrm{EAR} = (1 + \mathrm{MR})^{12} - 1$$

**where MR = monthly interest rate*

Effective Interest Rates

Example:

Given a monthly rate of 1%, what is the effective annual rate (EAR)? What is the annual percentage rate (APR)?

Effective Interest Rates

Example:

Given a monthly rate of 1%, what is the effective annual rate (EAR)? What is the annual percentage rate (APR)?

$$\text{EAR} = (1+.01)^{12} - 1 = r$$

$$\text{EAR} = (1+.01)^{12} - 1 = .1268 \text{ or } 12.68\%$$

$$\text{APR} = .01 \times 12 = .12 \text{ or } 12.00\%$$