

## Topics Covered

- Using The Present Value Formula to Value Bonds
- How Bond Prices Vary With Interest Rates
- The Term Structure of Interest Rates
- Explaining the Term Structure
- Real and Nominal Rates of Interest
- The Risk of Default
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## Bonds

## Terminology

- Bond - Security that obligates the issuer to make specified payments to the bondholder.
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- Face value (par value or principal value) Payment at the maturity of the bond. $\qquad$
- Coupon - The interest payments made to the bondholder.
- Coupon rate - Annual interest payment, as a percentage of face value.


## Bonds

## WARNING

The coupon rate IS NOT the discount rate used in the present value calculations.
-The coupon rate merely tells us what cash flow the bond will produce
oSince the coupon rate is listed as a \%, this misconception is quite common

## Valuing a Bond

The price of a bond is the present value of all cash flows generated by the bond (i.e. coupons and face value) discounted at the required rate of return

$$
\mathrm{PV}=\frac{\mathrm{cpn}}{(1+r)^{1}}+\frac{\mathrm{cpn}}{(1+r)^{2}}+\ldots .+\frac{(\mathrm{cpn}+\mathrm{par})}{(1+r)^{t}}
$$

Note: "cpn" is commonly used as an abbreviation for "coupon"

## Valuing a Bond

## Example - France

In October 2014 you purchase 100 euros of bonds in France which pay a $4.25 \%$ coupon every year. If the bond matures in 2018 and the YTM is $0.15 \%$, what is the value of the bond?

$$
\begin{aligned}
P V & =\frac{4.25}{1.0015}+\frac{4.25}{(1.0015)^{2}}+\frac{4.25}{(1.0015)^{3}}+\frac{104.25}{(1.0015)^{4}} \\
& =116.34 \text { euros }
\end{aligned}
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## Valuing a Bond as an Annuity

$P V($ bond $)=P V($ annuity of coupons $)+P V($ principal $)$

PV $($ bond $)=(\mathrm{cpn} \times \mathrm{PVAF})+($ final payment $\times$ discount factor $)$

$$
=4.25 \times\left[\frac{1}{.0015}-\frac{1}{.0015(1+.0015)^{4}}\right]+\frac{100}{(1+.0015)^{4}}
$$

$$
=116.34
$$

## Valuing a Bond

## Example

If today is October 1, 2015, what is the value of the following bond? An IBM Bond pays $\$ 115$ every September 30 for 5 years. In September 2020 it pays an additional \$1000 and retires the bond. The bond is rated AAA (WSJ AAA YTM is 7.5\%)
$\mathrm{PV}=\frac{115}{1.075}+\frac{115}{(1.075)^{2}}+\frac{115}{(1.075)^{3}}+\frac{115}{(1.075)^{4}}+\frac{1,115}{(1.075)^{5}}$

$$
=\$ 1,161.84
$$

## Valuing a Bond

## Example

What is the price of a 7.25 \% annual coupon bond, with a $\$ 1,000$ face value, which matures in 3 years?
Assume a required return of $0.35 \%$.
$\mathrm{PV}=\frac{72.50}{(1.0035)^{1}}+\frac{72.50}{(1.0035)^{2}}+\frac{1,072.50}{(1.0035)^{3}}$
$\mathrm{PV}=\$ 1,205.56$
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## Valuing a Bond

## Example (continued)

What is the price of a $7.25 \%$ annual coupon bond, with a $\$ 1,000$ face value, which matures in 3 years? Assume a required return of $0.35 \%$.

Bond prices are quoted as a percentage of par.

Par value $\times$ price $\%=\$$ price
$\$ 1,000 \times$ price $\%=\$ 1,205.56$ price $\%=120.56 \%$

## Valuing a Bond

Q: How did the calculation change, given semi-annual coupons versus annual coupon payments?

Twice as many payments, cut in half, over the same time period.


## Valuing a Bond

Example - USA
In November 2014 you purchase a 3 year US Government bond. The bond has an annual coupon rate of $4.25 \%$, paid semi-annually. If investors demand a $0.965 \%$ semiannual return, what is the price of the bond?
$\mathrm{PV}=\frac{21.25}{1.004825}+\frac{21.25}{(1.004825)^{2}}+\frac{21.25}{(1.004825)^{3}}+\frac{21.25}{(1.004825)^{4}}+\frac{21.25}{(1.004825)^{5}}+\frac{1021.25}{(1.004825)^{6}}$
= $\$ 1,096.90$

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## Treasury Yields

The interest rate on 10-year U.S. Treasury bonds 1900-2012


## Bond Rates of Return

Rate of Return - Total income per period per dollar invested

Rate of return $=\frac{\text { total income }}{\text { investment }}$

Rate of return $=\frac{\text { coupon income }+ \text { price change }}{\text { investment }}$

## Bond Rates of Return

## Example

A bond increases in price from $\$ 963.80$ to $\$ 1,380.50$ and pays a coupon of $\$ 21.875$ during the same period. What is the rate of return?

Rate of return $=\frac{21.875+(1380.50-963.80)}{963.80}=.455$ $\mathrm{ROR}=45.5 \%$
Bond Rates of Return
Bond Rates of Return

$\frac{\text { Example }}{\text { A bond increases in price from } \$ 963.80 \text { to } \$ 1,380.50}$| and pays a coupon of $\$ 21.875$ during the same period. |
| :--- |
| What is the rate of return? |

Rate of return $=\frac{21.875+(1380.50-963.80)}{963.80}=.455$
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| Duration |  |  |  |  |
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| Example |  |  |  |  |
| Calculate the duration of our $67 / 8 \%$ bond @ 4.9\% YTM |  |  |  |  |
| Year | CF | PV@YTM | \% of Total PV | $\% \times$ Year |
| 1 | 68.75 | 65.54 | . 060 | 0.060 |
| 2 | 68.75 | 62.48 | . 058 | 0.115 |
| 3 | 68.75 | 59.56 | . 055 | 0.165 |
| 4 | 68.75 | 56.78 | . 052 | 0.209 |
| 5 | 1068.75 | 841.39 | . 775 | 3.875 |
|  |  | 1085.74 | 1.00 Dur | 4.424 |



## Yield Curve

U.S. Treasury Strip Spot Rates as of November 2014


## Law of One Price

- All interest bearing instruments are priced to fit the term structure
- This is accomplished by modifying the asset price
- The modified price creates a new yield, which fits the term structure
- The new yield is called the yield to maturity (YTM)


## Example

\$1,000 Treasury bond expires in 5 years. Pays coupon
rate of $10.5 \%$. What is YTM if market price is 107.88 ?

| $\boldsymbol{C}_{0}$ | $\boldsymbol{C}_{1}$ | $\boldsymbol{C}_{\mathbf{2}}$ | $\boldsymbol{C}_{\mathbf{3}}$ | $\boldsymbol{C}_{\mathbf{4}}$ | $\boldsymbol{C}_{5}$ |
| :---: | :--- | :--- | :--- | :--- | :--- |
| -1078.80 | 105 | 105 | 105 | 105 | 1105 |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
| Calculate IRR $=\mathbf{8 . 5 \%}$ |  |  |  |  |  |

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## Explaining the Term Structure

Expectations Theory

- Term structure and capital budgeting $\checkmark$ CF should be discounted using term structure info
$\checkmark$ When rate incorporates all forward rates, use spot rate that equals project term $\checkmark$ Take advantage of arbitrage


## Debt \& Interest Rates

- Classical Theory of Interest Rates (Economics) - Developed by Irving Fisher
- Nominal Interest Rate = The rate you actually pay when you borrow money $\qquad$
- Real Interest Rate = The theoretical rate you pay when you borrow money, as determined by supply and demand


Annual U.S. Inflation Rates, 1900-2014

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Global Inflation Rates, 1900-2014


## Interest Rates \& Inflation

- In the presence of inflation, an investor's real interest rate is always less than the nominal interest rate

$$
1+\text { real rate }=\frac{1+\text { nominal rate }}{1+\text { inflation rate }}
$$

## Interest Rates \& Inflation

## Example

If you invest in a security that pays 10\% interest annually and inflation is 6\%, what is your real interest rate?

$$
1+\text { real rate }=\frac{1.10}{1.06}
$$

Real interest rate $=.03774$ or $3.774 \%$
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## Interest Rates \& Inflation

Treasury Inflation Protected Securities (TIPS)

Example
If you invest in 5\% coupon, 3 year TIPS and inflation is $3 \%$ each year, what are your real annual cash flows?

| Year | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: |
| Real cash flows | $\$ 50$ | $\$ 50$ | $\$ 1,050$ |

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## U.S. TIPS Bond Yields

Govt. Bills vs. Inflation, 1953-2014


Govt. Bills vs. Inflation, 1953-2014


## Default Risk

- Default or Credit Risk - The risk that a bond issuer may default on its bonds
- Default premium - The additional yield on a bond that investors require for bearing credit risk
- Investment grade - Bonds rated Baa or above by Moody's or BBB or above by Standard \& Poor's
- Junk bonds - Bond with a rating below Baa or BBB
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## Sovereign Bonds and Default Risk

- Sovereign Bonds and Default Risk
-Foreign currency debt
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$\checkmark$ Default occurs when foreign government borrows dollars
$\checkmark$ If crisis occurs, governments may run out of taxing capacity and default
$\checkmark$ Affects bond prices, yield to maturity


## Sovereign Bonds and Default Risk

- Sovereign Bonds and Default Risk
-Own currency debt
$\checkmark$ Less risky than foreign currency debt
$\checkmark$ Governments can print money to repay bonds
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## Sovereign Bonds and Default Risk

- Sovereign Bonds and Default Risk -Eurozone debt
$\checkmark$ Can't print money to service domestic debts
$\checkmark$ Money supply controlled by European Central Bank
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