# VALUING BONDS

# Topics Covered

- Using The Present Value Formula to Value Bonds
- How Bond Prices Vary With Interest Rates
- The Term Structure of Interest Rates
- Explaining the Term Structure
- Real and Nominal Rates of Interest
- The Risk of Default

# Bonds

Terminology

- Bond - Security that obligates the issuer to make specified payments to the bondholder.
- Face value (par value or principal value) - Payment at the maturity of the bond.
- Coupon - The interest payments made to the bondholder.
- Coupon rate - Annual interest payment, as a percentage of face value.

## Bonds

### WARNING

**The <u>coupon rate</u> IS NOT the <u>discount rate</u> used in the present value calculations.**

- The coupon rate merely tells us what cash flow the bond will produce
- Since the coupon rate is listed as a %, this misconception is quite common

# Valuing a Bond

The price of a bond is the present value of all cash flows generated by the bond (i.e. coupons and face value) discounted at the required rate of return

$$\text{PV} = \frac{\text{cpn}}{(1+r)^1} + \frac{\text{cpn}}{(1+r)^2} + \dots + \frac{(\text{cpn} + \text{par})}{(1+r)^t}$$

*Note:* “cpn” is commonly used as an abbreviation for “coupon”

# Valuing a Bond

***Example - France***

*In October 2014 you purchase 100 euros of bonds in France which pay a 4.25% coupon every year. If the bond matures in 2018 and the YTM is 0.15%, what is the value of the bond?*

$$\text{PV} = \frac{4.25}{1.0015} + \frac{4.25}{(1.0015)^2} + \frac{4.25}{(1.0015)^3} + \frac{104.25}{(1.0015)^4}$$

$$= 116.34 \text{ euros}$$

# Valuing a Bond as an Annuity

PV(bond) = PV(annuity of coupons) + PV(principal)

$$\begin{aligned}\text{PV (bond)} &= (\text{cpn} \times \text{PVAF}) + (\text{final payment} \times \text{discount factor})\\ &= 4.25 \times \left[\frac{1}{.0015} - \frac{1}{.0015(1+.0015)^4}\right] + \frac{100}{(1+.0015)^4}\\ &= 116.34\end{aligned}$$

# Valuing a Bond

***Example***

*If today is October 1, 2015, what is the value of the following bond? An IBM Bond pays $115 every September 30 for 5 years. In September 2020 it pays an additional $1000 and retires the bond. The bond is rated AAA (WSJ AAA YTM is 7.5%)*

$$\text{PV} = \frac{115}{1.075} + \frac{115}{(1.075)^2} + \frac{115}{(1.075)^3} + \frac{115}{(1.075)^4} + \frac{1{,}115}{(1.075)^5}$$

$$= \$1{,}161.84$$

# Valuing a Bond

***Example***

*What is the price of a 7.25 % annual coupon bond, with a $1,000 face value, which matures in 3 years? Assume a required return of 0.35%.*

$$PV = \frac{72.50}{(1.0035)^1} + \frac{72.50}{(1.0035)^2} + \frac{1{,}072.50}{(1.0035)^3}$$

$$PV = \$1{,}205.56$$

# Valuing a Bond

***Example (continued)***

*What is the price of a 7.25 % annual coupon bond, with a $1,000 face value, which matures in 3 years? Assume a required return of 0.35%.*

Bond prices are quoted as a percentage of par.

$$\text{Par value} \times \text{price \%} = \$\ \text{price}$$
$$\$1{,}000 \times \text{price \%} = \$1{,}205.56$$
$$\text{price \%} = 120.56\ \%$$

# Valuing a Bond

*Q: How did the calculation change, given semi-annual coupons versus annual coupon payments?*

Twice as many payments, cut in half, over the same time period.

# Valuing a Bond

***Example - USA***

In November 2014 you purchase a 3 year US Government bond. The bond has an annual coupon rate of 4.25%, paid semi-annually. If investors demand a 0.965% semiannual return, what is the price of the bond?

$$PV = \frac{21.25}{1.004825} + \frac{21.25}{(1.004825)^2} + \frac{21.25}{(1.004825)^3} + \frac{21.25}{(1.004825)^4} + \frac{21.25}{(1.004825)^5} + \frac{1021.25}{(1.004825)^6}$$

$$= \$1{,}096.90$$

# Treasury Yields

## Bond Rates of Return

Rate of Return – Total income per period per dollar invested

$$\text{Rate of return} = \frac{\text{total income}}{\text{investment}}$$

$$\text{Rate of return} = \frac{\text{coupon income} + \text{price change}}{\text{investment}}$$

# Bond Rates of Return

***Example***

*A bond increases in price from $963.80 to $1,380.50 and pays a coupon of $21.875 during the same period. What is the rate of return?*

$$\text{Rate of return} = \frac{21.875 + (1380.50 - 963.80)}{963.80} = .455$$

ROR = 45.5%

# Duration Formula

$$\text{Duration} = \frac{1 \times \text{PV}(C_1)}{\text{PV}} + \frac{2 \times \text{PV}(C_2)}{\text{PV}} + \frac{3 \times \text{PV}(C_3)}{\text{PV}} + \ldots + \frac{T \times \text{PV}(C_T)}{\text{PV}}$$

$$\text{Modified duration} = \text{volatility } (\%) = \frac{\text{duration}}{1 + \text{yield}}$$

# Duration Calculation

| | Year ($t$) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | |
| Payment | $90 | $90 | $90 | $90 | $90 | $90 | $ 1,090 | |
| PV($C_t$) at 4% | $86.54 | $83.21 | $80.01 | $76.93 | $73.97 | $71.13 | $ 828.31 | PV = $1,300.10 |
| Fraction of total value [PV($C_t$) / PV] | 0.0666 | 0.0640 | 0.0615 | 0.0592 | 0.0569 | 0.0547 | 0.6371 | |
| Year × fraction of total value [$t$ × PV($C_t$) / PV] | 0.0666 | 0.1280 | 0.1846 | 0.2367 | 0.2845 | 0.3283 | 4.4598 | Total = duration = 5.69 |

# Duration Calculation

| Year | $C_t$ | PV($C_t$) at 5.0% | Proportion of Total Value [PV($C_t$)/$V$] | Proportion of Total Value Time |
|---|---|---|---|---|
| 1 | 100 | 95.24 | 0.084 | 0.084 |
| 2 | 100 | 90.7 | 0.08 | 0.16 |
| 3 | 1100 | 950.22 | 0.836 | 2.509 |
| | | $V$ = 1136.16 | 1 | Duration= 2.753 years |

# Duration

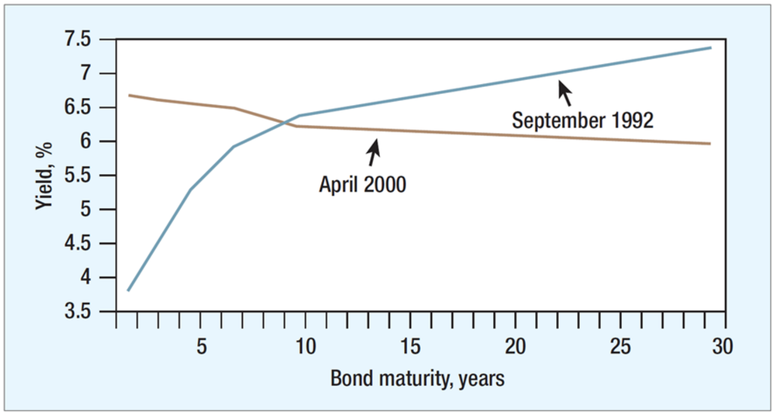
***Example***

*Calculate the duration of our 6 $^{7}/_{8}$% bond @ 4.9% YTM*

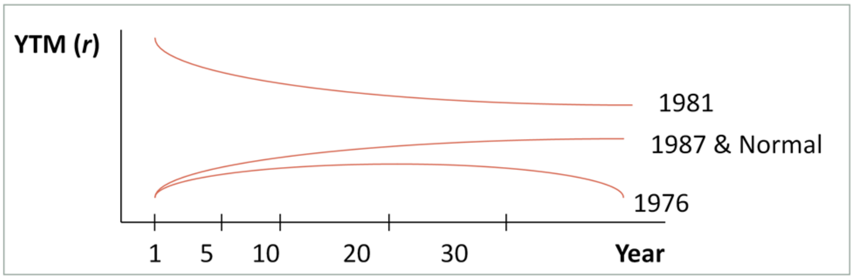
| Year | CF | PV@YTM | % of Total PV | % × Year |
|---|---|---|---|---|
| 1 | 68.75 | 65.54 | .060 | 0.060 |
| 2 | 68.75 | 62.48 | .058 | 0.115 |
| 3 | 68.75 | 59.56 | .055 | 0.165 |
| 4 | 68.75 | 56.78 | .052 | 0.209 |
| 5 | 1068.75 | 841.39 | .775 | 3.875 |
| | | 1085.74 | 1.00 | **Duration 4.424** |

# Interest Rates

- Short- and long-term interest rates do not always move in parallel. Between September 1992 and April 2000, U.S. short-term rates rose sharply while long term rates declined.



# Term Structure of Interest Rates




Spot Rate - The actual interest rate today ($t = 0$)

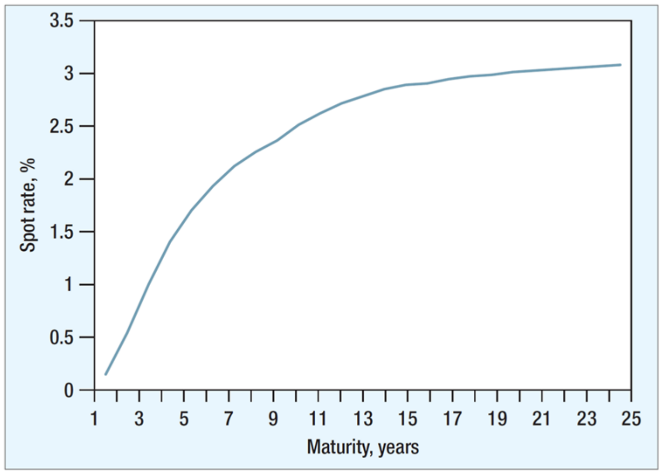
Forward Rate - The interest rate, fixed today, on a loan made in the future at a fixed time

Future Rate - The spot rate that is expected in the future

Yield To Maturity (YTM) - The IRR on an interest bearing instrument

# Yield Curve

U.S. Treasury Strip Spot Rates as of November 2014



## Law of One Price

- All interest bearing instruments are priced to fit the term structure
- This is accomplished by modifying the asset price
- The modified price creates a new yield, which fits the term structure
- The new yield is called the yield to maturity (YTM)

# Yield to Maturity

***Example***

*$1,000 Treasury bond expires in 5 years. Pays coupon rate of 10.5%. What is YTM if market price is 107.88?*

| $C_0$ | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ |
|---|---|---|---|---|---|
| −1078.80 | 105 | 105 | 105 | 105 | 1105 |

**Calculate IRR = 8.5%**



There are two types of problems in bond valuation:

- Given the yield to maturity calculate the price of the bond;
- Given the price of a bond calculate the yield to maturity.

Here is an example where the price of the bond is given and you are asked to calculate the yield to maturity.

Face value = $1,000; maturity = 5 years; coupon rate = 10.5%; price of the bond = $1,078.80, (107.88% of the face value). Calculate the yield to maturity of the bond.

PMT = 105; N = 5; PV = -1078.80; FV = 1,000.

Compute I = 8.5%.

IRR function in the calculator can also be used for calculating the YTM of the bond.

[Use the cash-flow register. $CF_0$ = -1078.80; $C_1$ = 105, $F_1$ = 4; $C_2$ = 1105, $F_2$ = 1; IRR→Compute→8.5%.]

## Explaining the Term Structure

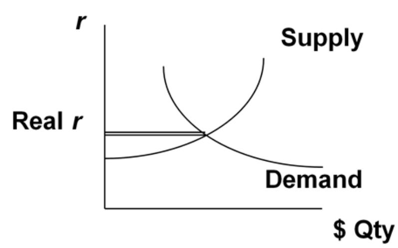
Expectations Theory

- Term structure and capital budgeting
  - ✓CF should be discounted using term structure info
  - ✓When rate incorporates all forward rates, use spot rate that equals project term
  - ✓Take advantage of arbitrage

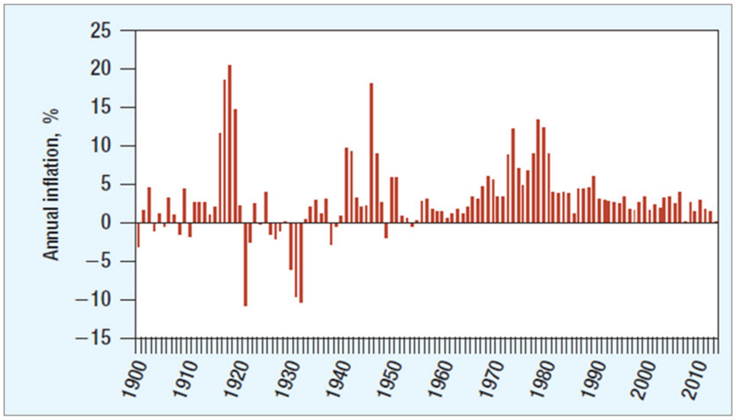
Many theories try to explain the shape of the yield curve. The most popular theory is the unbiased expectations theory. This theory states that forward interest rates are unbiased estimates of expected future spot rates. Term structure implies that for capital budgeting CF should be discounted to include term structure information. The spot rate used for discounting cash flows should be equal to the term of the project.

# Debt & Interest Rates

- Classical Theory of Interest Rates (Economics)
  - Developed by Irving Fisher
- Nominal Interest Rate = The rate you actually pay when you borrow money
- Real Interest Rate = The theoretical rate you pay when you borrow money, as determined by supply and demand

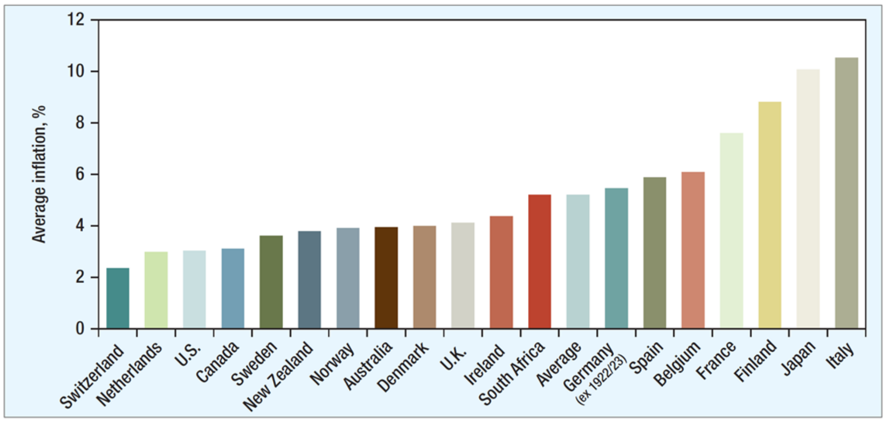


# Annual U.S. Inflation Rates, 1900-2014




Annual U.S. inflation rates from 1900–2011 are shown.

# Global Inflation Rates, 1900-2014




The U.S. has, on average, low inflation.

# Interest Rates & Inflation

- In the presence of inflation, an investor's **real** interest rate is always less than the **nominal** interest rate

$$1 + \text{real rate} = \frac{1 + \text{nominal rate}}{1 + \text{inflation rate}}$$

## Interest Rates & Inflation

***Example***

*If you invest in a security that pays 10% interest annually and inflation is 6%, what is your real interest rate?*

$$1 + \text{real rate} = \frac{1.10}{1.06}$$

Real interest rate = .03774 or 3.774%

# Interest Rates & Inflation

Treasury Inflation Protected Securities (TIPS)

***Example***

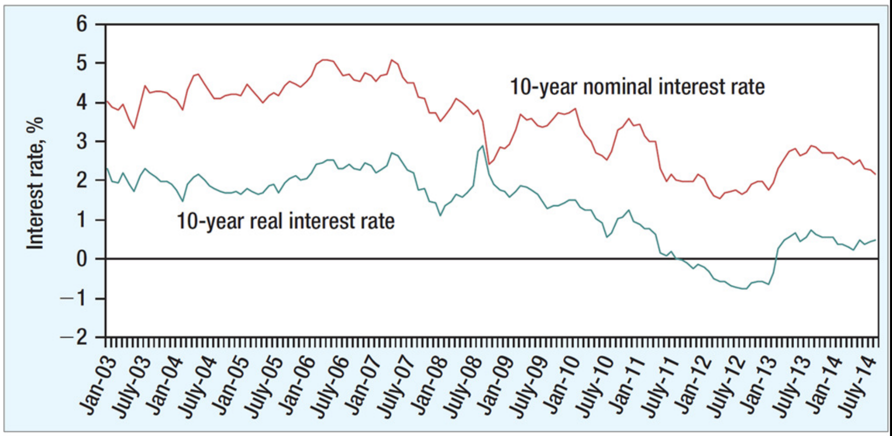
*If you invest in 5% coupon, 3 year TIPS and inflation is 3% each year, what are your **real** annual cash flows?*

| Year | 1 | 2 | 3 |
|---|---|---|---|
| Real cash flows | **$50** | **$50** | **$1,050** |

**TIPS:** U.S. Treasury issued debt with fixed real flows, but with nominal cash flows (interest and principal) that are increased as the consumer price index increases.

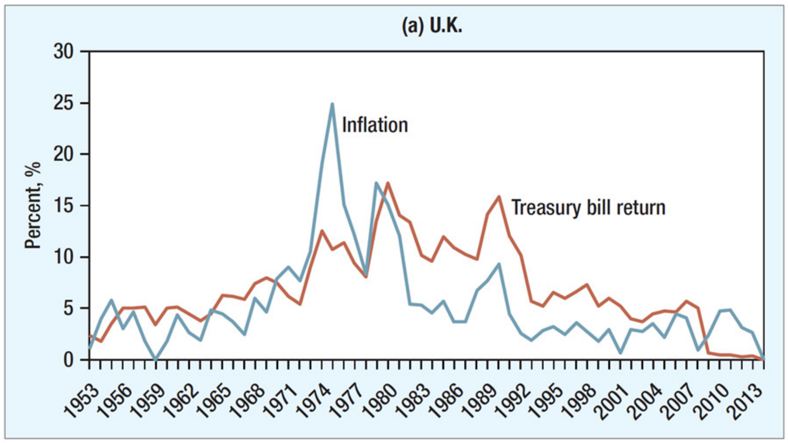
Note: What are the nominal cash flows for the example above?

# U.S. TIPS Bond Yields



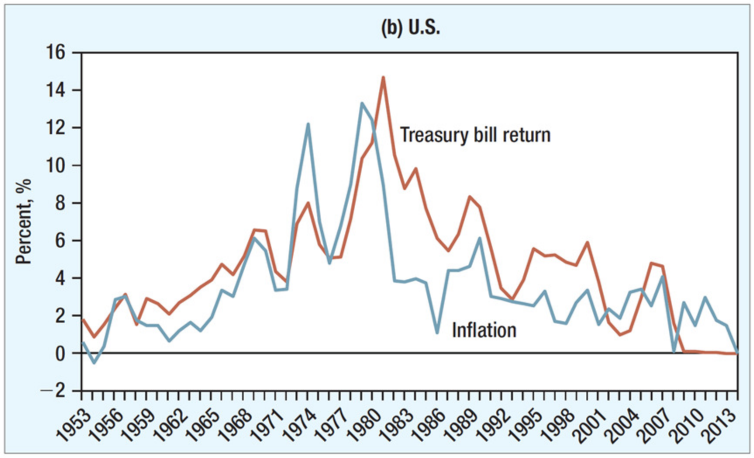

The graph shows the nominal and real interest rates over time for the UK.

# Govt. Bills vs. Inflation, 1953-2014



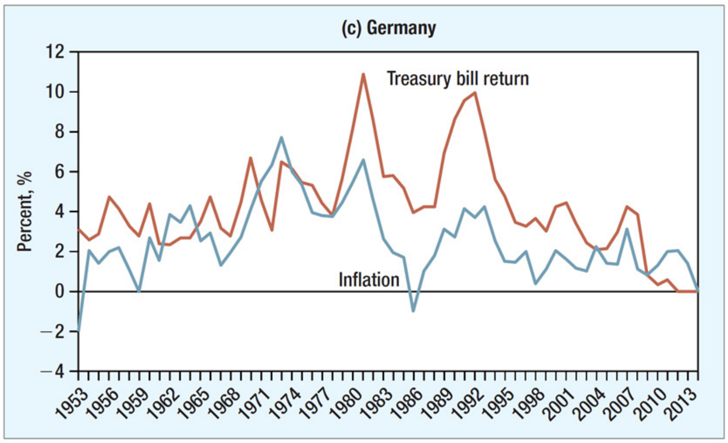

The graph shows the T-bill rates and inflation over time for the UK.

# Govt. Bills vs. Inflation, 1953-2014




The graph shows the T-bill rates and inflation over time for the U.S.

## Govt. Bills vs. Inflation, 1953-2014




The graph shows the T-bill rates and inflation over time for Germany.

## Default Risk

- Default or Credit Risk - The risk that a bond issuer may default on its bonds
- Default premium - The additional yield on a bond that investors require for bearing credit risk
- Investment grade - Bonds rated Baa or above by Moody's or BBB or above by Standard & Poor's
- Junk bonds - Bond with a rating below Baa or BBB

# Default Risk

| Moody' s | Standard & Poor's | Safety |
|---|---|---|
| Aaa | AAA | The strongest rating; ability to repay interest and principal is very strong. |
| Aa | AA | Very strong likelihood that interest and principal will be repaid |
| A | A | Strong ability to repay, but some vulnerability to changes in circumstances |
| Baa | BBB | Adequate capacity to repay; more vulnerability to changes in economic circumstances |
| Ba | BB | Considerable uncertainty about ability to repay. |
| B | B | Likelihood of interest and principal payments over sustained periods is questionable. |
| Caa | CCC | Bonds in the Caa/CCC and Ca/CC classes may already be |
| Ca | CC | in default or in danger of imminent default |
| C | C | C-rated bonds offer little prospect for interest or principal on the debt ever to be repaid. |

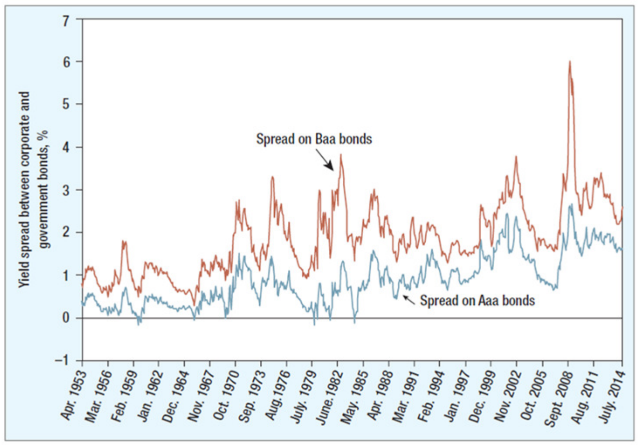
# Prices and Yields of Corporate Bonds

| Issuer Name | Coupon (%) | Maturity | S&P Rating | Price (%) | Yield (%) |
|---|---|---|---|---|---|
| Johnson & Johnson | 3.55 | 2021 | AAA | 108.35 | 2.16 |
| Walmart | 4.25 | 2021 | AA | 110.44 | 2.48 |
| Alabama Power | 3.95 | 2021 | A | 105.84 | 2.97 |
| Dow Chemical | 8.85 | 2021 | BBB | 132.39 | 3.49 |
| Rosetta Restaurants | 5.625 | 2021 | BB | 97.00 | 6.20 |
| Elizabeth Arden | 7.375 | 2021 | B | 96.25 | 8.14 |
| Alpha Natural Resources | 6.25 | 2021 | CCC | 50.20 | 20.37 |

Sample listing of corporate bonds and the corresponding yields.

# Yield Spread

Yield Spreads between Corporate and 10-year Treasury Bonds



## Sovereign Bonds and Default Risk

- Sovereign Bonds and Default Risk
  - Foreign currency debt
    - ✓Default occurs when foreign government borrows dollars
    - ✓If crisis occurs, governments may run out of taxing capacity and default
    - ✓Affects bond prices, yield to maturity

This slide introduces foreign currency debt, which is the most common type of debt to cause countries to default. The book includes real-world examples of foreign currency default.

## Sovereign Bonds and Default Risk

- Sovereign Bonds and Default Risk
  - Own currency debt
    - ✓Less risky than foreign currency debt
    - ✓Governments can print money to repay bonds

This slide introduces own currency debt, which is less likely to cause countries to default. The book includes real-world examples of own currency default.

## Sovereign Bonds and Default Risk

- Sovereign Bonds and Default Risk
  - Eurozone debt
    - ✓Can't print money to service domestic debts
    - ✓Money supply controlled by European Central Bank

This slide examines the monetary policies of the Eurozone, the member countries of which cannot generate own-currency debt. Eurozone countries have ceded control of their money supply to the European Central Bank.