

## Topics Covered

- Over a Century of Capital Market History in One Easy Lesson
- Measuring Portfolio Risk
- Calculating Portfolio Risk
- How Individual Securities Affect Portfolio Risk
- Diversification \& Value Additivity


## The Value of an Investment of $\$ 1$ in 1899



## The Value of an Investment of $\$ 1$ in 1899

Real Returns


## Average Market Risk Premiums (by country)



## Dividend Yield

Dividend yields in the U.S. 1900-2014


## Rates of Return 1900-2011

Stock Market Index Returns


## Measuring Risk

Histogram of Annual Stock Market Returns (1900-2014)


## Measuring Risk

## Variance

- Average value of squared deviations from mean
- A measure of volatility


## Standard Deviation

- Average value of squared deviations from mean
- A measure of volatility


## Measuring Risk

## Coin Toss Game-calculating variance and standard deviation



## Measuring Risk



## Equity Market Risk (by country)

Average Risk (1900-2014)


## Dow Jones Risk

## 7-13

Annualised Standard Deviation of the DJIA over the preceding 52 weeks (1900-2014)


## Measuring Risk

Diversification - Strategy designed to reduce risk by spreading the portfolio across many investments.

Unique Risk - Risk factors affecting only that firm. Also called "diversifiable risk."

Market Risk - Economy-wide sources of risk that affect the overall stock market. Also called "systematic risk."

## Comparing Returns

The value of a portfolio evenly divided between Newmont Mining and Ford was less volatile than either stock on its own. The assumed initial investment is $\$ 100$.


## Measuring Risk



## Measuring Risk



## Portfolio Risk

The variance of a two stock portfolio is the sum of these four boxes

|  | Stock 1 | Stock 2 |
| :--- | :---: | :---: |
| Stock 1 | $\mathrm{x}_{1}^{2} \sigma_{1}^{2}$ | $\mathrm{x}_{1} \mathrm{x}_{2} \sigma_{12}=$ |
|  | $\mathrm{x}_{1} \mathrm{x}_{2} \rho_{12} \sigma_{1} \sigma_{2}$ |  |
| Stock 2 | $\mathrm{x}_{1} \mathrm{x}_{2} \sigma_{12}=$ | $\mathrm{x}_{2}^{2} \sigma_{2}^{2}$ |

## Portfolio Risk

## Example

Suppose you invest $60 \%$ of your portfolio in JNJ and $40 \%$ in Ford. The expected dollar return on your JNJ is $8.0 \%$ and on Ford is $18.8 \%$. The expected return on your portfolio is:

Expected return $=(.60 \times 8.0)+(.40 \times 18.8)=12.3 \%$

## Portfolio Risk

## Example

Suppose you invest $60 \%$ of your portfolio in JNJ and $40 \%$ in Ford. The expected dollar return on your JNJ is 8.0\% and on Ford is $18.8 \%$. The standard deviation of their annualised daily returns are 13.2\% and 31.0\%, respectively. Assume a correlation coefficient of 1.0 and calculate the portfolio variance.

|  | JNJ | Ford |
| :--- | :---: | :---: |
| JNJ | $x_{1}^{2} \sigma_{1}^{2}=(.60)^{2} \times(13.2)^{2}$ | $x_{1} x_{2} \rho_{12} \sigma_{1} \sigma_{2}=.40 \times .60$ <br> $\times 1 \times 13.2 \times 31.0$ |
| Ford | $x_{1} x_{2} \rho_{12} \sigma_{1} \sigma_{2}=.40 \times .60$ <br> $\times 1 \times 13.2 \times 31.0$ | $x_{2}^{2} \sigma_{2}^{2}=(.40)^{2} \times(31.0)^{2}$ |

## Portfolio Risk

## Example

Suppose you invest $60 \%$ of your portfolio in JNJ and $40 \%$ in Ford. The expected dollar return on your JNJ is 8.0\% and on Ford is $18.8 \%$. The standard deviation of their annualized daily returns are $13.2 \%$ and $31.0 \%$, respectively. Assume a correlation coefficient of 1.0 and calculate the portfolio variance.

Portfolio variance $=\left[(.60)^{2} \times(13.2)^{2}\right]$

$$
\begin{aligned}
& +\left[(.40)^{2} \times(31.0)^{2}\right] \\
& +2(.40 \times .60 \times 1 \times 13.2 \times 31.0)=412.90
\end{aligned}
$$

Standard deviation $=\sqrt{412.90}=20.3 \%$

## Portfolio Risk

## Example

Suppose you invest 60\% of your portfolio in JNJ and 40\% in Ford. The expected dollar return on your JNJ is 8.0\% and on Ford is $18.8 \%$. The standard deviation of their annualized daily returns are 13.2\% and 31.0\%, respectively. Assume a correlation coefficient of .019 and calculate the portfolio variance.

Portfolio variance $=\left[(.60)^{2} \times(13.2)^{2}\right]$

$$
+\left[(.40)^{2} \times(31.0)^{2}\right]
$$

$$
+2(.40 \times .60 \times 0.19 \times 13.2 \times 31.0)=253.80
$$

Standard deviation $=\sqrt{253.80}=15.90 \%$

## Portfolio Risk

## Another Example

Suppose you invest 60\% of your portfolio in JNJ and 40\% in Ford. The expected dollar return on your JNJ is 8.0\% and on Ford is $18.8 \%$. The standard deviation of their annualized daily returns are $13.2 \%$ and $31.0 \%$, respectively. Assume a correlation coefficient of -1.00 and calculate the portfolio variance.

$$
\begin{aligned}
\text { Portfolio variance }= & {\left[(.60)^{2} \times(13.2)^{2}\right] } \\
& +\left[(.40)^{2} \times(31.0)^{2}\right] \\
& +2(.40 \times .60 \times(-1.00) \times 13.2 \times 31.0)=20.1
\end{aligned}
$$

Standard deviation $=\sqrt{20.10}=4.50 \%$

## Portfolio Risk

Expected portfolio return $=\left(x_{1} r_{1}\right)+\left(x_{2} r_{2}\right)$

Portfolio variance $=x_{1}^{2} \sigma_{1}^{2}+x_{2}^{2} \sigma_{2}^{2}+2\left(x_{1} x_{2} \rho_{12} \sigma_{1} \sigma_{2}\right)$

## Portfolio Risk

| Example | Correlation Coefficient $=.4$ |  |  |
| :--- | :--- | :---: | :---: |
| Stocks | $\sigma$ | $\%$ of Portfolio | Avg Return |
| ABC Corp | 28 | $60 \%$ | $15 \%$ |
| Big Corp | 42 | $40 \%$ | $21 \%$ |

Standard deviation $=$ weighted $\mathrm{avg}=\underline{33.6}$
Standard deviation $=$ Portfolio $=\underline{28.1}$

Real standard deviation:

$$
\begin{aligned}
& =\left(28^{2}\right)\left(.6^{2}\right)+\left(42^{2}\right)\left(.4^{2}\right)+2(.4)(.6)(28)(42)(.4) \\
& =\underline{28.1} \text { CORRECT }
\end{aligned}
$$

Return : $r=(15 \%)(.60)+(21 \%)(.4)=17.4 \%$

## Portfolio Risk

| Example |  | Correlation Coefficient $=.4$ |  |
| :--- | :--- | :---: | :---: |
| Stocks | $\sigma$ | $\%$ of Portfolio | Avg Return |
| ABC Corp | 28 | $60 \%$ | $15 \%$ |
| Big Corp | 42 | $40 \%$ | $21 \%$ |

Standard deviation = weighted avg = $\underline{33.6}$
Standard deviation $=$ portfolio $=\underline{28.1}$
Return $=$ weighted avg = portfolio $=\underline{17.4 \%}$
Let's add New Corp. stock to the portfolio

## Portfolio Risk

| Example | Correlation Coefficient $=.3$ <br> \% of Portfolio |  |  |
| :--- | :--- | ---: | :---: |
| Stocks | $\sigma$ | $50 \%$ | Avg Return |
| Portfolio | 28.1 | $50 \%$ | $17.4 \%$ |
| New Corp | 30 |  | $19 \%$ |

NEW standard deviation = weighted avg $=31.80$
NEW standard deviation $=$ portfolio $=\underline{23.43}$
NEW return $=$ weighted avg $=$ portfolio $=\underline{18.20 \%}$

NOTE: Higher return \& Lower risk How did we do that?

DIVERSIFICATION

## Portfolio Risk

The shaded boxes contain variance terms; the remainder contain covariance terms.


## To calculate portfolio variance add up the boxes

## Portfolio Risk, Beta

Market Portfolio - Portfolio of all assets in the economy. In practice a broad stock market index, such as the S\&P Composite, is used to represent the market.

Beta - Sensitivity of a stock's return to the return on the market portfolio.

## Portfolio Risk, Beta

The return on Ford stock changes on average by
$1.44 \%$ for each additional 1\% change in the market return. Beta is therefore 1.44.


## Portfolio Risk, Beta

The green line shows that a well diversified portfolio of randomly selected stocks ends up with $\beta=1$ and a standard deviation equal to the market's-in this case 20\%. The upper red line shows that a well diversified portfolio with $\beta=1.5$ has a standard deviation of about $30 \%-1.5$ times that of the market. The lower blue line shows that a
 well-diversified portfolio with $\beta$ $=.5$ has a standard deviation of about $10 \%$-half that of the market.

## Portfolio Risk, Beta

$$
B_{i}=\frac{\sigma_{i m}}{\sigma_{m}^{2}} \longrightarrow \begin{aligned}
& \text { Covariance with the } \\
& \text { market }
\end{aligned}
$$

Variance of the market

## Portfolio Risk, Beta

Calculating the variance of the market returns and the covariance between the returns on the market and those of Anchovy Queen.
Beta is the ratio of the variance to the covariance (i.e., $\beta=\sigma_{i m} / \sigma^{2}{ }_{m}$ )

| 1 | (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 |  |  |  |  |  |  | Product of |
| 3 |  |  |  | Deviation | Deviation | Squared | deviations |
| 4 |  |  |  | from | from average | deviation | from average |
| 5 |  | Market | Anchovy 0 | average | Anchovy Q | from average | returns |
| 6 | Month | return | return | market return | return | market return | (cols $4 \times 5$ ) |
| 7 | 1 | -8\% | -11\% | - 10 | -13 | 100 | 130 |
| 8 | 2 | 4 | 8 | 2 | 6 | 4 | 12 |
| 9 | 3 | 12 | 19 | 10 | 17 | 100 | 170 |
| 10 | 4 | -6 | -13 | -8 | -15 | 64 | 120 |
| 11 | 5 | 2 | 3 | 0 | 1 | 0 | 0 |
| 12 | 6 | 8 | 6 | 6 | 4 | 36 | 24 |
| 13 | Average | 2 | 2 |  | Total | 304 | 456 |
| 14 |  |  |  | Variance $=\sigma_{m}^{2}=304 / 6=50.67$ |  |  |  |
| 15 |  |  |  | Covariance $=\sigma_{\text {im }}=456 / 6=76$ |  |  |  |
| 16 |  |  |  | Beta $(\beta)=\sigma_{i m} / \sigma_{m}^{2}=76 / 50.67=1.5$ |  |  |  |

