

#### PORTFOLIO THEORY AND THE CAPITAL ASSET PRICING MODEL



# **Topics Covered**

- Harry Markowitz and the Birth of Portfolio
  Theory
- The Relationship between Risk and Return
- Validity and the Role of the CAPM
- Some Alternative Theories

- Combining stocks into portfolios can reduce standard deviation, below the level obtained from a simple weighted average calculation
- Correlation coefficients make this possible
- The various weighted combinations of stocks that create this standard deviations constitute the set of *efficient portfolios*

Price changes vs. Normal distribution

IBM - Daily % change 1994-2013



Standard Deviation vs. Expected Return



Standard Deviation vs. Expected Return



Standard Deviation vs. Expected Return



Expected returns and standard deviations vary given different weighted combinations of the stocks



			Three Efficient Portfolios—Percentages Allocated to Each Stock (%)		
	Expected Return (%)	Standard Deviation (%)	А	В	C
Caterpillar	17.6	29.2	0	0	
Microsoft	12.5	20.7	13	11	
Consolidated Edison	8.3	13.8	27	22	
Newmont	18.1	30.0	11	18	
Apple	15.3	25.2	4	8	
Johnson & Johnson	8.0	13.2	10	0	
Campbell Soup	1.0	16.6	15	17	
Walmart	9.3	15.3	17	10	
Ford	18.8	31.0	2	9	100
Dow Chemical	16.7	27.5	3	5	
Expected portfolio return			10.61	12.84	18.8
Portfolio standard deviation			9.14	10.35	31.0

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Three efficient portfolios all from the same 10 stocks



• Each half egg shell represents the possible weighted combinations for two stocks.

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 The composite of all stock sets constitutes the efficient frontier



Standard Deviation

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Lending or borrowing at the risk free rate  $(r_f)$  allows us to exist outside the efficient frontier.



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<u>Example</u>		Correlation coefficient = .19		
<u>Stocks</u>	σ	% of Portfolio	Avg Return	
JNJ	13.2	60%	15%	
Ford	31.0	40%	21%	

Standard deviation = weighted avg = 20.3%Standard deviation = portfolio = 15.9%Return = weighted avg = portfolio = 12.3%

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<u>Example</u>		Correlation coefficie	nt = .4
<u>Stocks</u>	$\sigma$	% of Portfolio	Avg Return
ABC Corp	28	60%	15%
Big Corp	42	40%	21%

Standard deviation = weighted avg = 33.6%Standard deviation = portfolio = 28.1%Return = weighted avg = portfolio = 17.4%

Additive standard deviation (common sense):

= .28 (60%) + .42 (40%) = <u>33.6% WRONG</u>

Real standard deviation:  $= \sqrt{x_1^2 \sigma_1^2 + x_2^2 \sigma_2^2 + 2(x_1 x_2 \rho_{12} \sigma_1 \sigma_2)}$   $= \sqrt{.60^2 \cdot .28^2 + .40^2 \cdot .42^2 + 2(.6)(.4)(.4)(.28)(.42)}$  = .281 or 28.1% CORRECT

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<u>Previous Example</u>		Correlation coefficient = $.3$		
<u>Stocks</u>	σ	% of Portfolio	Avg Return	
Portfolio	28.1	50%	17.4%	
New Corp	30	50%	19%	

NEW standard deviation = weighted avg = 31.80 %NEW standard deviation = portfolio = 23.43 %NEW return = weighted avg = portfolio = 18.20%

NOTE: Higher return & lower risk How did we do that? <u>DIVERSIFICATION</u>

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The ratio of the risk premium to the standard deviation is called the Sharpe ratio: Goal is to move up and left. WHY?

Sharpe ratio = 
$$\frac{r_p - r_f}{\sigma_p}$$

Return

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Low Risk	High Risk
High Return	High Return
Low Risk Low Return	High Risk Low Return

Risk

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Risk











SML Equation = 
$$r_f + \beta(r_m - r_f)$$

#### Capital Asset Pricing Model

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$$r = r_f + \beta(r_m - r_f)$$

CAPM

# **Expected Returns**

These estimates of the returns expected by investors in November 2014 were based on the capital asset pricing model. We assumed 2% for the interest rate  $r_f$  and 7% for the expected risk premium  $r_m - r_f$ .

		Expected Return
Stock	Beta (β)	$\mathbf{r}_f = \beta(\mathbf{r}_m - \mathbf{r}_f)$
Caterpillar	1.66	13.6
Dow Chemical	1.65	13.5
Ford	1.44	12.1
Microsoft	0.98	8.9
Apple	0.91	8.4
Johnson & Johnson	0.53	5.7
Walmart	0.45	5.2
Campbell Soup	0.39	4.7
Consolidated Edison	0.17	3.2
Newmont	0	2.0

# SML Equilibrium

In equilibrium no stock can lie below the security market line. For example, instead of buying stock A, investors would prefer to lend part of their money and put the balance in the market portfolio. And instead of buying stock B, they would prefer to borrow and invest in the market portfolio.



## Testing the CAPM

Beta vs. Average Risk Premium



#### Testing the CAPM

Beta vs. Average Risk Premium



# Testing the CAPM

Return vs. Book-to-Market

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http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data\_library.html

# Arbitrage Pricing Theory

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#### **Alternative to CAPM**

Return = 
$$a + b_1(r_{factor1}) + b_2(r_{factor2}) + b_3(r_{factor3}) + \dots + noise$$

Expected risk premium =  $r - r_f$ =  $b_1(r_{\text{factor1}} - r_f) + b_2(r_{\text{factor2}} - r_f) + \dots$ 

# Arbitrage Pricing Theory

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# Estimated risk premiums for taking on risk factors (1978-1990)

Factor	Estimated Risk Premium		
Factor	$(r_{\text{factor}} - r_f)$		
Yield spread	5.10%		
Interest rate	61		
Exchange rate	59		
Real GNP	.49		
Inflation	83		
Market	6.36		

# Three Factor Model

Steps to Identify Factors

- 1. Identify a reasonably short list of macroeconomic factors that could affect stock returns
- 2. Estimate the expected risk premium on each of these factors ( $r_{\text{factor 1}} r_{f_{,}}$  etc.)
- 3. Measure the sensitivity of each stock to the factors  $(b_1, b_2, \text{ etc.})$

# **Three Factor Model**

	Three-Factor Model				CAPM
	Factor Sensitivities				-
	<b>b</b> <sub>market</sub>	<b>b</b> <sub>size</sub>	<b>b</b> <sub>book-to-market</sub>	Expected return <sup>a</sup>	Expected return <sup>b</sup>
Autos	1.37	0.62	-0.07	13.4%	12.7%
Banks	1.12	0.02	0.74	13.5	10.6
Chemicals	1.35	0.05	-0.19	10.7	11.3
Computers	1.17	-0.10	-0.33	8.3	9.7
Construction	1.13	0.82	0.57	15.5	12.1
Food	0.52	-0.15	0.00	5.1	5.4
Oil and gas	1.21	-0.20	0.02	9.9	10.1
Pharmaceuticals	0.77	-0.27	-0.31	5.0	4.9
Telecoms	0.87	-0.08	0.04	8.0	8.0
Utilities	0.48	-0.16	0.08	5.2	5.2