

# Maths 190 (Math'1 Models in Finance)

## Assignment No. 1

**Due Date: 14:30, 09 February 2017**

**Venue: B6**

NOTE: Only problems marked with ♣ are required for submission; there are 4 required problems in this problem set (see pages 3-6). The other problems are practice exercises.

### GUIDELINES ON SUBMITTING ASSIGNMENTS

- Your assignment paper must include the Marking Scheme as a cover page. This marking scheme can be downloaded from the course website. Failure to follow this instruction can result to a 2-point deduction on your assignment mark.
- Do not submit your rough work! Do the problem set and then re-write it at least once - neatly, with adequate amount of clear explanation. The rewriting stage is the most important one for finding errors in one's work, and it will also deepen your understanding of the subject matter. Assignments are marked for both technical correctness and elegance of presentation.
- Bear in mind to include a sufficient amount of explanation about your work so that any marker does not have to guess what you mean. The grader of your work will determine if you understand what you are writing, not merely that you reach the particular correct answer.
- On questions where a computer output is required, include the output in the text of your answer at the appropriate locations - do not put it all in a bunch at the end of your assignment. Unless, you are instructed to submit your work in a CD or disc, you are expected to hand in a PRINTED COPY. ***Assignments sent via e-mail will not be accepted.***
- YOU MUST WRITE YOUR OWN WORK IN YOUR OWN WORDS, using full sentences and proper English grammar. It is your responsibility to familiarise yourself with the provisions of the University Regulation concerning academic integrity and honesty. ***Any behaviour that can potentially lead to plagiarism and cheating (including copying from/sharing with another student answers in assignments and exams) is a serious offence and carries with it severe penalty.***

Problems below, unless indicated otherwise, were taken from Hull, J. (2012), *Options, Futures, and Other Derivatives*, 8<sup>th</sup> edition, Prentice Hall, Boston.  
**N.B.:** *For problems not required for submission, you will learn and benefit more if you attempt solving them first before looking at their solutions.*

### Problem 1.3

What is the difference between (a) entering into a long futures contract when the futures price is \$50 and (b) taking a long position in a call option with a strike price of \$50?

### Problem 1.9

You would like to speculate on a rise in the price of a certain stock. The current stock price is \$29 and a three-month call with a strike price of \$30 costs \$2.90. You have \$5,800 to invest. Identify two alternative strategies. Briefly outline the advantages and disadvantages of each.

### Problem 1.11

When first issued, a stock provides funds for a company. Is the same true of an exchange-traded stock option? Discuss.

### Additional Problem 1

A cattle farmer expects to have 120,000 pounds of live cattle to sell in three months. The live-cattle futures contract on the Chicago Mercantile Exchange is for the delivery of 40,000 pounds of cattle. How can the farmer use the contract for hedging? From the farmer's viewpoint, what are the pros and cons of hedging?

### Problem 1.13

Suppose that a March call option on a stock with a strike price of \$50 costs \$2.50 and is held until March. Under what circumstances will the holder of the option make a gain? Under what circumstances will the option be exercised? Draw a diagram showing how the profit on a long position in the option depends on the stock price at the maturity of the option.

### Problem 1.14

Suppose that a June put option on a stock with a strike price of \$60 costs \$4 and is held until June. Under what circumstances will the holder of the option make a gain? Under what circumstances will the option be exercised? Draw a diagram showing how the profit

on a short position in the option depends on the stock price at the maturity of the option.

### **Problem 1.15**

It is May and a trader writes a September call option with a strike price of \$20. The stock price is \$18, and the option price is \$2. Describe the investor's cash flows if the option is held until September and the stock price is \$25 at that time.

### **Problem 1.16**

A trader writes a December put option with a strike price of \$30. The price of the option is \$4. Under what circumstances does the trader make a gain?

### **Additional Problem 2**

An airline executive has argued: "There is no point in our using oil futures. There is just as much chance that the price of oil in the future will be less than the futures price as there is that it will be greater than this price." Discuss the executive's viewpoint.

### **Problem 1.18**

A United States company expects to have to pay 1 million Canadian dollars in six months. Explain how the exchange rate risk can be hedged using (a) a forward contract; (b) an option.

### **Problem 1.19**

A trader enters into a short forward contract on 100 million yen. The forward exchange rate is \$0.0080 per yen. How much does the trader gain or lose if the exchange rate at the end of the contract is (a) \$0.0074 per yen; (b) \$0.0091 per yen?

### **♣ Required Assignment Question 1 (Continuation of Example given in the 01-Feb-2017 lecture) [4 points]**

The forward price of a bond for a contract with a delivery date in one year is \$905. The current spot price is \$900. Coupon payments of \$40 are expected in 6 months and one year. The 6-month and one-year risk-free interest rates are 9% p.a. and 10% p.a., respectively. Determine the arbitrage opportunity (if any).

**♣ Required Assignment Question 2 (Problem 5.28) [4 points]**

A bank offers a corporate client a choice between borrowing cash at 11% per annum and borrowing gold at 2% per annum. (If gold is borrowed, interest must be repaid in gold. Thus, 100 ounces borrowed today would require 102 ounces to be repaid in one year.) The risk-free interest rate is 9.25% per annum, and storage costs are 0.5% per annum. Discuss whether the rate of interest on the gold loan is too high or too low in relation to the rate of interest on the cash loan. The interest rates on the two loans are expressed with annual compounding. The risk-free interest rate and storage costs are expressed with continuous compounding.

**Problem 3.1**

Under what circumstances are (a) a short hedge and (b) a long hedge appropriate?

**Problem 3.5**

Give three reasons why the treasurer of a company might not hedge the company's exposure to a particular risk.

**Problem 5.9**

A one-year long forward contract on a non-dividend-paying stock is entered into when the stock price is \$40 and the risk-free rate of interest is 10% per annum with continuous compounding.

- a) What are the forward price and the initial value of the forward contract?
- b) Six months later, the price of the stock is \$45 and the risk-free interest rate is still 10%. What are the forward price and the value of the forward contract?

**Problem 5.12**

Suppose that the risk-free interest rate is 10% per annum with continuous compounding and that the dividend yield on a stock index is 4% per annum. The index is standing at 400, and the futures price for a contract deliverable in four months is 405. What arbitrage opportunities does this create?

**Problem 5.14**

The two-month interest rates in Switzerland and the United States are 2% and 5% per annum, respectively, with continuous compounding. The spot price of the Swiss franc is

\$0.8000. The futures price for a contract deliverable in two months is \$0.8100. What arbitrage opportunities does this create?

### Problem 5.16

Suppose that  $F_1$  and  $F_2$  are two futures contracts on the same commodity with times to maturity,  $t_1$  and  $t_2$ , where  $t_2 > t_1$ . Prove that

$$F_2 \leq F_1 e^{r(t_2 - t_1)}$$

where  $r$  is the interest rate (assumed constant) and there are no storage costs. For the purposes of this problem, assume that a futures contract is the same as a forward contract.

### Problem 5.19

Show that the growth rate in an index futures price equals the excess return of the portfolio underlying the index over the risk-free rate. Assume that the risk-free interest rate and the dividend yield are constant.

### ♣ Required Assignment Question 3 (Not in Hull, 8<sup>th</sup> ed) [8 points]

In each case below provide a table (and plot) showing the relationship between profit/loss and final stock price. Ignore the impact of discounting.

- Call options with strike prices of \$25 and \$30 cost \$7.90 and \$4.18, respectively. Both have maturity of six months. Demonstrate the profit/loss pattern for a trading strategy (called bull spread) of buying the \$25 call and selling the \$30 call. Why would an investor be engaging such a strategy? [2.5 pts]
- Put options with strike prices of \$25, \$30, and \$35 cost \$0.70, \$2.14 and \$4.57, respectively. All these options have maturity of one year. Demonstrate the profit/loss pattern for a trading strategy (called butterfly spread) of buying one \$25 put option, buying one \$35 put option and selling 2 \$30 put option. Why would an investor be engaging such a strategy? [3 pts]
- A six-month call option with a strike price of \$35 costs \$1.85. A six-month put option with a strike price of \$25 costs \$0.28. Demonstrate the profit/loss pattern for a trading strategy (called strangle) of buying the call and put options. Why would an investor be engaging such a strategy? [2.5 pts]

### Problem 9.1

An investor buys a European put on a share for \$3. The stock price is \$42 and the strike price is \$40. Under what circumstances does the investor make a profit? Under what

circumstances will the option be exercised? Draw a diagram showing the variation of the investor's profit with the stock price at the maturity of the option.

### Problem 9.12

A trader buys a call option with a strike price of \$45 and a put option with a strike price of \$40. Both options have the same maturity. The call costs \$3 and the put costs \$4. Draw a diagram showing the variation of the trader's profit with the asset price.

### ♣ Required Assignment Question 4 (Cvitanic and Zapatero, problem 18 of chapter 1) [4 points]

At time zero you enter into a short position in futures contract on 20 shares of stock XYZ at the futures price of \$50.00. Moreover, you write (sell) 5 "exotic" options of the following type: they are put options, but using as the underlying asset the average of today's stock price and the stock price at maturity of the underlying asset. The option's strike price is \$52, the option selling price today is \$5.00 per option and today's stock price is \$49.00 per share. The maturity of all your positions is  $T =$  months. What is your total profit or loss two months from now if

- (a) at maturity the price of one stock share is \$57.00? [2 pts]
- (b) at maturity the price of one stock share is \$47.00? [2 pts]

### Problem 9.13

Explain why an American option is always worth at least as much as a European option on the same asset with the same strike price and exercise date.

### Problem 9.14

Explain why an American option is always worth at least as much as its intrinsic value.

### Problem 9.15

Explain carefully the difference between writing a put option and buying a call option.

### Problem 10.1

List the six factors affecting option prices.

### Problem 10.4

Give two reasons that the early exercise of an American call option on a non-dividend paying stock is not optimal. The first reason should involve the time value of money. The second reason should apply even if interest rates are zero.

**Problem 10.5**

“The early exercise of an American put is a trade-off between the time value of money and the insurance value of a put”. Explain this statement.

**Problem 10.8**

Explain why the arguments leading to put–call parity for European options cannot be used to give a similar result for American options.

**Problem 10.10**

What is a lower bound for the price of a two-month European put option on a non-dividend-paying stock when the stock price is \$58, the strike price is \$65, and the risk-free interest rate is 5% per annum?

**Problem 10.11**

A four-month European call option on a dividend-paying stock is currently selling for \$5. The stock price is \$64, the strike price is \$60, and a dividend of \$0.80 is expected in one month. The risk-free interest rate is 12% per annum for all maturities. What opportunities are there for an arbitrageur?

**Problem 10.14**

The price of a European call that expires in six months and has a strike price of \$30 is \$2. The underlying stock price is \$29, and a dividend of \$0.50 is expected in two months and again in five months. The term structure is flat, with all risk-free interest rates being 10%. What is the price of a European put option that expires in six months and has a strike price of \$30?

**Problem 10.15**

Explain carefully the arbitrage opportunities in Problem 10.14 if the European put price is \$3.

**Problem 10.16**

The price of an American call on a non-dividend-paying stock is \$4. The stock price is \$31, the strike price is \$30, and the expiration date is in three months. The risk-free interest rate is 8%. Derive upper and lower bounds for the price of an American put on the same stock with the same strike price and expiration date.

**Problem 10.17**

Explain carefully the arbitrage opportunities in Problem 10.16 if the American put price is greater than the calculated upper bound.

**Problem 10.19**

Prove the result in equation (10.11). (Hint: For the first part of the relationship consider (a) a portfolio consisting of a European call plus an amount of cash equal to  $D + K$  and (b) a portfolio consisting of an American put option plus one share.)

Result in (10.11) is stated as:  $C - P \geq S_0 - D - K$