Maths 190 – 2^{nd} Semester, AY 2016-17
Solutions to Assignment No. 1
(Questions assigned for submission)

♣Required Assignment Question 1 (Continuation of Example given in the 01-Feb-2017 lecture) [4 points]

The forward price of a bond for a contract with a delivery date in one year is \$905. The current spot price is \$900. Coupon payments of \$40 are expected in 6 months and one year. The 6-month and one-year risk-free interest rates are 9% p.a. and 10% p.a., respectively. Determine the arbitrage opportunity (if any).

Answer: The PV of the income is $I = 38.23 + 40e^{-0.1} = \$74.4235$. The correct forward price as shown in class is $F = (900 – 74.4235)\, e^{0.1} = \$912.40$. Therefore the forward price of \$905 is too low relative to the correct price of \$912.40. 0.5 pts

An investor who holds the bond can

(i) Sell one bond 0.5 pts

(ii) Enter into a long forward contract to repurchase the bond in one year. 0.5 pts

Of the \$900 realised from selling the bond, \$38.23 is invested for 6 months at 9% p.a. and \$861.77 is invested for 1 year at 10% p.a. 0.5 pts

This strategy produces a cash flow of \$40 at the 6-month point and a cash flow of \$952.40 at the one-year point. 0.5 pts

The \$40 replaces the coupon that would have been received on the bond at the six-month point. Of the \$952.40, \$40 replaces the coupon that would have been received on the bond at the one-year point. 0.5 pts

Under the terms of the forward contract, the bond is repurchased for \$905.00. 0.5 pts

The profit from the strategy of selling the bond and buying it back is, therefore, \$952.40 - \$40.00 - \$905.00 = \$7.40. 0.5 pts

♣Required Assignment Question 2 (Problem 5.28) [4 points]

A bank offers a corporate client a choice between borrowing cash at 11% per annum and borrowing gold at 2% per annum. (If gold is borrowed, interest must be repaid in gold. Thus, 100 ounces borrowed today *would require 102 ounces to be repaid in one year.) The risk-free interest rate is 9.25% per annum, and storage costs are 0.5% per annum. Discuss whether the rate of interest on the gold loan is too high or too low in relation to the rate of interest on the cash loan. The interest rates on the two loans are expressed with annual compounding. The risk-free interest rate and storage costs are expressed with continuous compounding.*

Answer: Suppose that the price of gold is \$1000 per ounce and the corporate client wants to borrow \$1,000,000. The client has a choice between borrowing \$1,000,000 in the usual way and borrowing 1,000 ounces of gold. 0.5 pts If it borrows \$1,000,000 in the usual way, an amount equal to $1,000,000 \times 1.11 = \$1,110,000$ 0.5 pts must be repaid. If it borrows 1,000 ounces of gold it must repay 1,020 ounces. 0.5 pts In equation (5.12) or formula derived in class, $r = 0.0925$ and $u = 0.005$ so that the forward price is

$$1000e^{(0.0925+0.005)\times 1} = 1102.41$$ 0.5 pts

By buying 1,020 ounces of gold in the forward market the corporate client can ensure that the repayment of the gold loan costs

$$1,020 \times 1102.41 = \$1,124,460$$ 0.5 pts

Clearly the cash loan is the better deal ($1,124,460 > 1,110,000$). 0.5 pts

This argument shows that the rate of interest on the gold loan is too high. What is the correct rate of interest? Suppose that R is the rate of interest on the gold loan. The client must repay $1,000(1+R)$ ounces of gold. When forward contracts are used the cost of this is

0.5 pts

$$1,000(1+R) \times 1102.41$$

This equals the \$1,110,000 required on the cash loan when $R = 0.688\%$ 0.5 pts. The rate of interest on the gold loan is too high by about 1.31%. However, this might be simply a reflection of the higher administrative costs incurred with a gold loan. ***It is interesting to note that this is not an artificial question. Many banks are prepared to make gold loans.***

♣Required Assignment Question 3 (Not in Hull, 8th ed) [8 points]

In each case below provide a table (and plot) showing the relationship between profit/loss and final stock price. Ignore the impact of discounting.

a. *Call options with strike prices of \$25 and \$30 cost \$7.90 and \$4.18, respectively. Both have maturity of six months. Demonstrate the profit/loss pattern for a trading strategy (called bull spread) of buying the \$25 call and selling the \$30 call. Why would an investor be engaging such a strategy? [2.5 pts]*

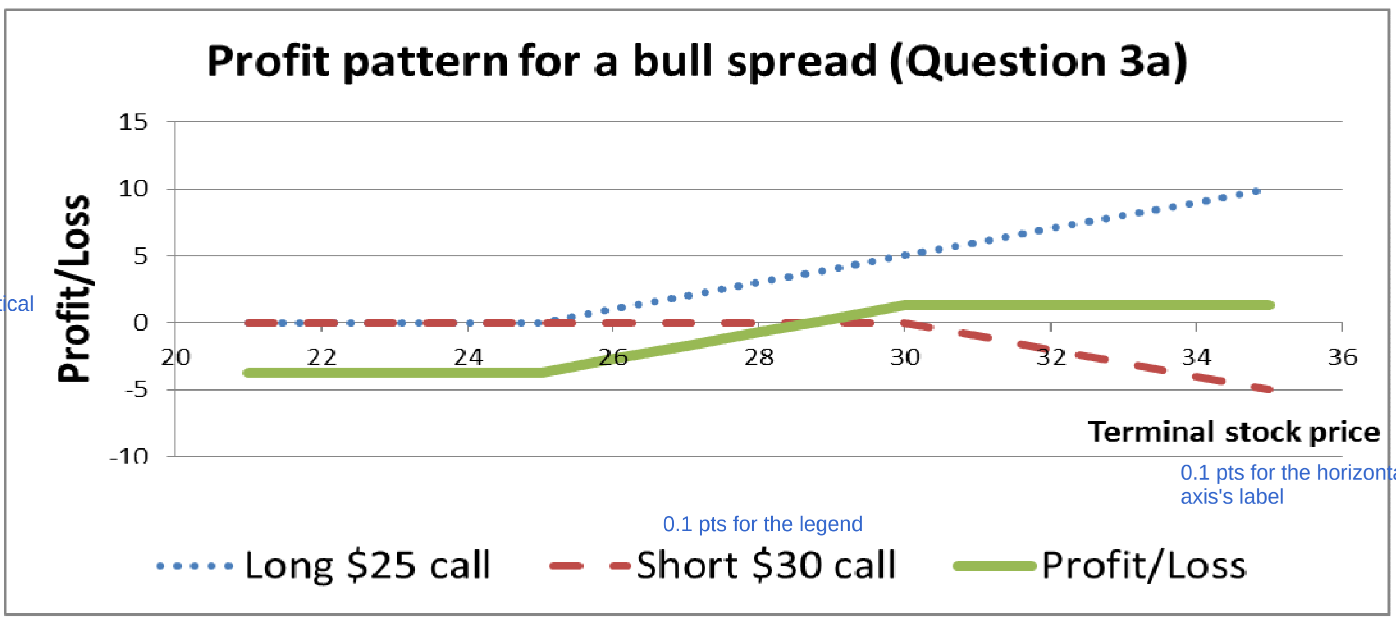
   Answer: A call option with a strike price of 25 costs \$7.90 and a call option with a strike price of 30 costs \$4.18. The cost of the bull spread is therefore $7.90 - 4.18 = 3.72$.

The profit/loss, ignoring the impact of discounting, is displayed below.

0.1 pts for each correct entry

Stock Price Range	*Long \$25 call*	*Short \$30 call*	*Total payoff*	*Profit/Loss*
$S_T \le 25$	max(S_T – 25,0) =0	- max(S_T – 30,0) =0	0	0-3.72 = –3.72
$25 < S_T < 30$	max(S_T – 25,0) = S_T – 25	- max(S_T – 30,0) =0	S_T – 25	S_T – 25 – 3.72 = $S_T - 28.72$
$S_T \ge 30$	max(S_T – 25,0) = S_T – 25	- max(S_T – 30,0) = 30 - S_T	30 - 25	30 – 25 – 3.72 = 1.28

1 pt for the correct profit pattern



0.1 pts for the vertical axis's label

0.1 pts for the horizontal axis's label

0.1 pts for the legend

b. *Put options with strike prices of $25, $30, and $35 cost $0.70, $2.14 and $4.57, respectively. All these options have maturity of one year. Demonstrate the profit/loss pattern for a trading strategy (called butterfly spread) of buying one $25 put option, buying one $35 put option and selling 2 $30 put options. Why would an investor be engaging such a strategy? [3 pts]*

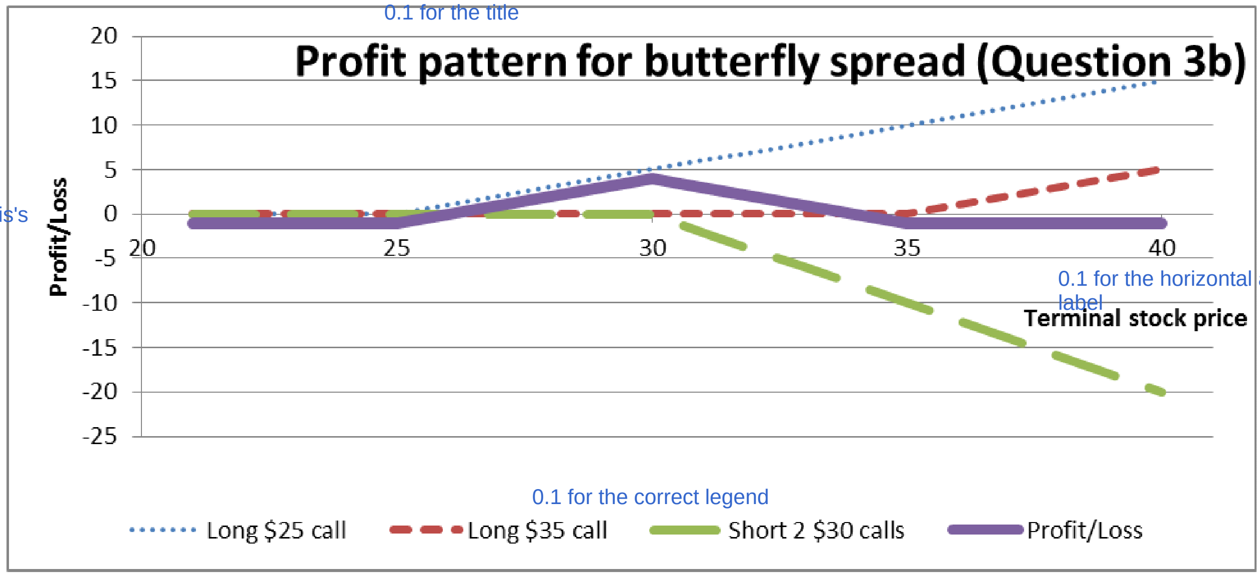
Answer: Put options with maturities of one year and strike prices of 25, 30, and 35 cost $0.70, $2.14, and $4.57, respectively. The cost of the butterfly spread is $1.00 (allowing for rounding errors) since $0.70+4.57-2\times2.14=0.99$. The profits, ignoring the impact of discounting, are given below.

0.4 for each correct row

Stock Price Range	*Long 1 $25 call*	*Long 1 $35 call*	*Short 2 $30calls*	*Total payoff*	*Profit/ Loss*
$S_T \le 25$	max(S_T – 25,0) = 0	max(S_T – 35,0) = 0	-2max(S_T – 30,0) = 0	0	–1.00
$25 < S_T < 30$	max(S_T – 25,0) = S_T – 25	max(S_T – 35,0) = 0	-2max(S_T – 30,0) = 0	S_T – 25	$S_T - 26.00$
$30 \le S_T < 35$	max(S_T – 25,0) = S_T – 25	max(S_T – 35,0) = 0	-2max(S_T – 30,0) = -2(S_T – 30)	S_T -25 – 2S_T +60 = 35- S_T	$34.00 - S_T$
$S_T \ge 35$	max(S_T – 25,0) = S_T – 25	max(S_T – 35,0) = S_T – 35	-2max(S_T – 30,0) = =-2(S_T – 30)	S_T -25 + S_T -35 – 2S_T +60 = 0	-1.00

1.2 for the correct profit pattern

0.1 for the vertical axis's label



c. A six-month call option with a strike price of $35 costs $1.85. A six-month put option with a strike price of $25 costs $0.28. Demonstrate the profit/loss pattern for a trading strategy (called strangle) of buying the call and put options. Why would an investor be engaging such a strategy? **[**2.5 pts**]**

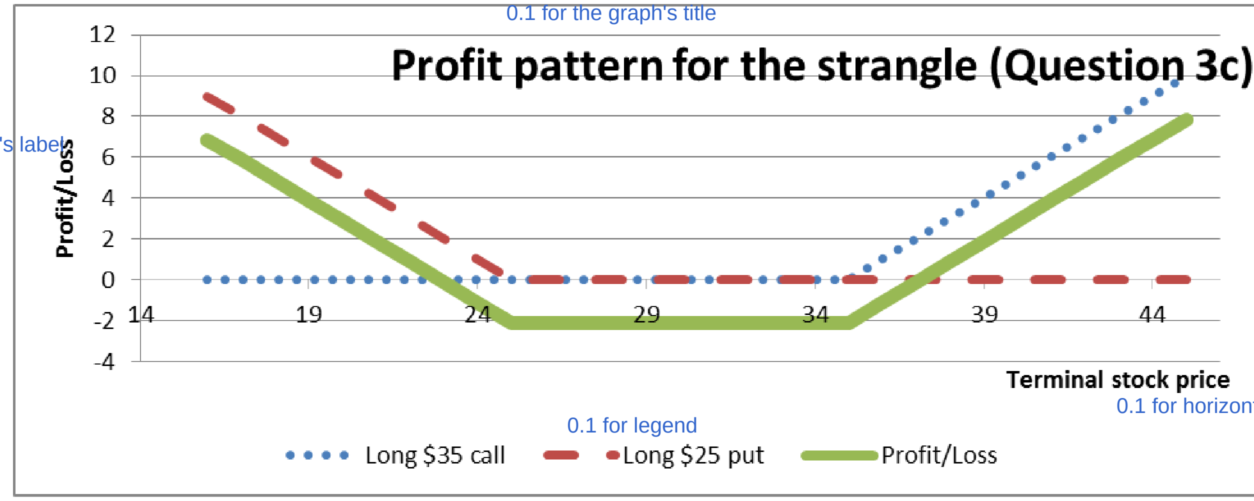
Answer: A six-month call option with a strike price of 35 costs 1.85. A six-month put option with a strike price of 25 costs 0.28. The cost of the strangle is therefore $1.85+0.28=2.13$. The profits ignoring the impact of discounting are

0.1 pts for each correct entry

Stock Price Range	*Long $35 call*	*Long $25 put*	*Total Payoff*	*Profit/Loss*
$S_T \le 25$	max(S_T – 35,0) = 0	max(25 - S_T, 0) = 25 - S_T	25 - S_T	$22.87-S_T$
$25 < S_T < 35$	max(S_T – 35,0) = 0	max(25 - S_T, 0) = 0	0	-2.13
$S_T \ge 35$	max(S_T – 35,0) = S_T – 35	max(25 - S_T, 0) = 0	S_T – 35	$S_T-37.13$

0.6 for the correct profit patterns

0.1 for vertical axis's label



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♣Required Assignment Question 4 (Cvitanic and Zapatero, problem 18 of chapter 1) [4 points]

At time zero you enter into a short position in futures contract on 20 shares of stock XYZ at the futures price of $50.00. Moreover, you write (sell) 5 "exotic" options of the following type: they are put options, but using as the underlying asset the average of today's stock price and the stock price at maturity of the underlying asset. The option's strike price is $52, the option selling price today is $5.00 per option and today's stock price is $49.00 per share. The maturity of all your positions is T = months. What is your total profit or loss two months from now if

(a) at maturity the price of one stock share is $57.00? [2 pts]

0.5 pts

Answer: You lose 20(57-50) = $140 in the futures contract. You receive 5(5) = $25.00 0.5 pts for the options. Since the average of the initial and the final stock price is $53, the put option is not exercised, that is, you have to pay $(5)(52-53)^+ = 0$ as the options' payoff. 0.5 pts Your total profit or loss is

P&L = -20(57-50) + 5(5) – 0 = - $115, which is a loss. 0.5 pts

(b) at maturity the price of one stock share is $47.00? [2 pts]

0.5 pts

Answer: You gain 20(50-47) = $60 from the futures contract. You receive %(5) = $25.00 for the options. 0.5 pts The average of the initial and the final stock price is $48, the put option is exercised, and you have to pay $5(52-48)^+ = \$20$ as the options' payoff. 0.5 pts Your total profit or loss is

P&L = 20(50-47) + 5(5) – $5(52-48)^+$ = $65, which is a gain. 0.5 pts