Maths 190 (2nd Semester, AY 2016-17)
Solutions to Assignment No. 1
(Only for questions not assigned for submission)

NOTE: *To be consistent with Hull's notation in the following exercises, option's strike price may also be denoted by K instead of X. If this is the case, it will be clear from the formulation and context of the question.*

Problem 1.3
What is the difference between (a) entering into a long futures contract when the futures price is $50 and (b) taking a long position in a call option with a strike price of $50?

Answer: In the first case the trader is obligated to buy the asset for $50. (The trader does not have a choice.) In the second case the trader has an option to buy the asset for $50. (The trader does not have to exercise the option.)

Problem 1.9
You would like to speculate on a rise in the price of a certain stock. The current stock price is $29 and a three-month call with a strike price of $30 costs $2.90. You have $5,800 to invest. Identify two alternative strategies. Briefly outline the advantages and disadvantages of each.

Answer: One strategy would be to buy 200 shares. Another would be to buy 2,000 options. If the share price does well the second strategy will give rise to greater gains. For example, if the share price goes up to $40 you gain $[2,000\times(\$40-\$30)]-\$5,800=\$14,200$ from the second strategy and only $200\times(\$40-\$29)=\$2,200$ from the first strategy. However, if the share price does badly, the second strategy gives greater losses. For example, if the share price goes down to $25, the first strategy leads to a loss of $200\times(\$29-\$25)=\$800,$ whereas the second strategy leads to a loss of the whole $5,800 investment. This example shows that options contain built in leverage.

Problem 1.11
When first issued, a stock provides funds for a company. Is the same true of an exchange-traded stock option? Discuss.

Answer: An exchange-traded stock option provides no funds for the company. It is a security sold by one investor to another. The company is not involved. By contrast, a stock when it is first issued is sold by the company to investors and does provide funds for the company.

Additional Problem 1

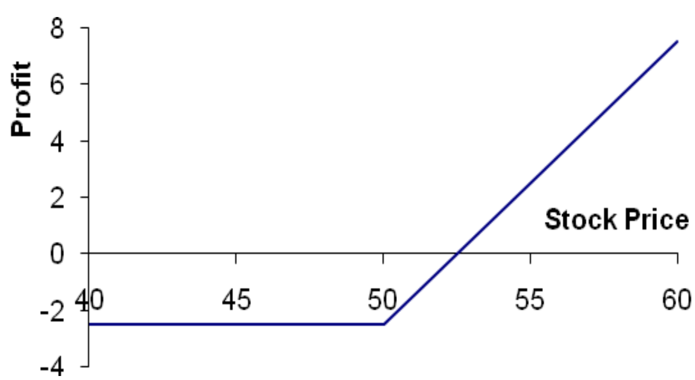
A cattle farmer expects to have 120,000 pounds of live cattle to sell in three months. The live-cattle futures contract on the Chicago Mercantile Exchange is for the delivery of 40,000 pounds of cattle. How can the farmer use the contract for hedging? From the farmer's viewpoint, what are the pros and cons of hedging?

Answer: The farmer can short 3 contracts that have 3 months to maturity. If the price of cattle falls, the gain on the futures contract will offset the loss on the sale of the cattle. If the price of cattle rises, the gain on the sale of the cattle will be offset by the loss on the futures contract. Using futures contracts to hedge has the advantage that it can at no cost reduce risk to almost zero. Its disadvantage is that the farmer no longer gains from favorable movements in cattle prices.

Problem 1.13

Suppose that a March call option on a stock with a strike price of $50 costs $2.50 and is held until March. Under what circumstances will the holder of the option make a gain? Under what circumstances will the option be exercised? Draw a diagram showing how the profit on a long position in the option depends on the stock price at the maturity of the option.

Answer: The holder of the option will gain if the price of the stock is above $52.50 in March. **(This ignores the time value of money.)** The option will be exercised if the price of the stock is above $50.00 in March. The profit as a function of the stock price is shown below.

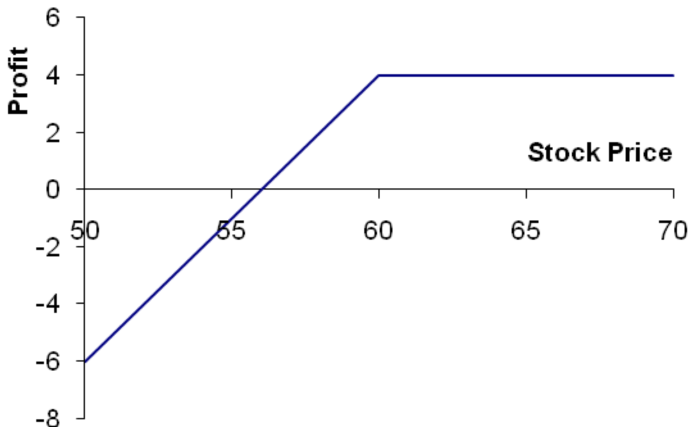


Profit from long position in Problem 1.13

Problem 1.14

Suppose that a June put option on a stock with a strike price of $60 costs $4 and is held until June. Under what circumstances will the holder of the option make a gain? Under what circumstances will the option be exercised? Draw a diagram showing how the profit on a short position in the option depends on the stock price at the maturity of the option.

Answer: The seller of the option will lose money if the price of the stock is below $56.00 in June. **(This ignores the time value of money.)** The option will be exercised if the price of the stock is below $60.00 in June. The profit as a function of the stock price is shown below.



Profit pattern for the short position in Problem 1.14

Problem 1.15

It is May and a trader writes a September call option with a strike price of $20. The stock price is $18, and the option price is $2. Describe the investor's cash flows if the option is held until September and the stock price is $25 at that time.

Answer: The trader has an inflow of $2 in May and an outflow of $5 in September. The $2 is the cash received from the sale of the option. The $5 is the result of the option being exercised. The investor has to buy the stock for $25 in September and sell it to the purchaser of the option for $20.

Problem 1.16

A trader writes a December put option with a strike price of $30. The price of the option is $4. Under what circumstances does the trader make a gain?

Answer: The trader makes a gain if the price of the stock is above $26 at the time of exercise. (This ignores the time value of money.)

Additional Problem 2

An airline executive has argued: "There is no point in our using oil futures. There is just as much chance that the price of oil in the future will be less than the futures price as there is that it will be greater than this price." Discuss the executive's viewpoint.

Answer: It may well be true that there is just as much chance that the price of oil in the future will be above the futures price as that it will be below the futures price. This means that the use of a futures contract for speculation would be like betting on whether a coin comes up heads or tails. But it might make sense for the airline to use futures for hedging rather than speculation. The futures contract then has the effect of reducing risks. It can be argued that an airline should not expose its shareholders to risks associated with the future price of oil when there are contracts available to hedge the risks.

Problem 1.18

A United States company expects to have to pay 1 million Canadian dollars in six months. Explain how the exchange rate risk can be hedged using (a) a forward contract; (b) an option.

Answer: The company could enter into a long forward contract to buy 1 million Canadian dollars in six months. This would have the effect of locking in an exchange rate equal to the current forward exchange rate. Alternatively the company could buy a call option giving it the right (but not the obligation) to purchase 1 million Canadian dollars at a certain exchange rate in six months. This would provide insurance against a strong Canadian dollar in six months while still allowing the company to benefit from a weak Canadian dollar at that time.

Problem 1.19

A trader enters into a short forward contract on 100 million yen. The forward exchange rate is $0.0080 per yen. How much does the trader gain or lose if the exchange rate at the end of the contract is (a) $0.0074 per yen; (b) $0.0091 per yen?

Answer:

a) The trader sells 100 million yen for $0.0080 per yen when the exchange rate is $0.0074 per yen. The gain is 100×0.0006 millions of dollars or $60,000.

b) The trader sells 100 million yen for $0.0080 per yen when the exchange rate is $0.0091 per yen. The loss is 100×0.0011 millions of dollars or $110,000.

♣Required Assignment Question 1 (Continuation of Example given in the 01-Feb-2017 lecture) [4 points]

The forward price of a bond for a contract with a delivery date in one year is $905. The current spot price is $900. Coupon payments of $40 are expected in 6 months and one year. The 6-month and one-year risk-free interest rates are 9% p.a. and 10% p.a., respectively. Determine the arbitrage opportunity (if any).

Answer: This question is an assigned problem for submission. Solution is in a separate sheet.

♣Required Assignment Question 2 (Problem 5.28) [4 points]

(a) A bank offers a corporate client a choice between borrowing cash at 11% per annum and borrowing gold at 2% per annum. (If gold is borrowed, interest must be repaid in gold. Thus, 100 ounces borrowed today would require 102 ounces to be repaid in one year.) The risk-free interest rate is 9.25% per annum, and storage costs are 0.5% per annum. Discuss whether the rate of interest on the gold loan is too high or too low in relation to the rate of interest on the cash loan. The interest rates on the two loans are expressed with annual compounding. The risk-free interest rate and storage costs are expressed with continuous compounding.

Answer: This question is an assigned problem for submission. Solution is in a separate sheet.

Problem 3.1

Under what circumstances are (a) a short hedge and (b) a long hedge appropriate?

Answer: A *short hedge* is appropriate when a company owns an asset and expects to sell that asset in the future. It can also be used when the company does not currently own the asset but expects to do so at some time in the future. A *long hedge* is appropriate when a company knows it will have to purchase an asset in the future. It can also be used to offset the risk from an existing short position.

Problem 3.5

Give three reasons why the treasurer of a company might not hedge the company's exposure to a particular risk.

Answer: (a) If the company's competitors are not hedging, the treasurer might feel that the company will experience less risk if it does not hedge. (b) The shareholders might not want the company to hedge because the risks are hedged within their portfolios. (c) If there is a loss on the hedge and a gain from the company's exposure to the underlying asset, the treasurer might feel that he or she will have difficulty justifying the hedging to other executives within the organisation.

Problem 5.9

A one-year long forward contract on a non-dividend-paying stock is entered into when the stock price is \$40 and the risk-free rate of interest is 10% per annum with continuous compounding.

a) What are the forward price and the initial value of the forward contract?
b) Six months later, the price of the stock is \$45 and the risk-free interest rate is still 10%. What are the forward price and the value of the forward contract?

Answer:

a) The forward price F is given $F = Se^{rT} = 40e^{(0.1)(1)} = 44.21$

or \$44.21. The initial value of the forward contract is zero.

b) The delivery price K in the contract is \$44.21. The value of the contract, f, after six months is given by $f = S - Ke^{-rT} = 45 - 44.21e^{(-0.1)(0.5)} = 2.95$

i.e., it is \$2.95. The forward price is $45e^{(0.1)(0.5)} = 47.31$

or \$47.31.

Problem 5.12

Suppose that the risk-free interest rate is 10% per annum with continuous compounding and that the dividend yield on a stock index is 4% per annum. The index is standing at 400, and the futures price for a contract deliverable in four months is 405. What arbitrage opportunities does this create?

Answer: The theoretical futures price is

$$400e^{(0.10-0.04)\times 4/12} = 408.08$$

The actual futures price is only 405. This shows that the index futures price is too low relative to the index. The correct arbitrage strategy is

1. Buy futures contracts
2. Short the shares underlying the index.

Problem 5.14

The two-month interest rates in Switzerland and the United States are 2% and 5% per annum, respectively, with continuous compounding. The spot price of the Swiss franc is \$0.8000. The futures price for a contract deliverable in two months is \$0.8100. What arbitrage opportunities does this create?

Answer: The theoretical futures price is

$$0.8000e^{(0.05-0.02)\times 2/12} = 0.8040$$

The actual futures price is too high. This suggests that an arbitrageur should buy Swiss francs and short Swiss francs futures.

Problem 5.16

Suppose that F_1 and F_2 are two futures contracts on the same commodity with times to maturity, t_1 and t_2, where $t_2 > t_1$. Prove that

$$F_2 \leq F_1 e^{r(t_2 - t_1)}$$

where r is the interest rate (assumed constant) and there are no storage costs. For the purposes of this problem, assume that a futures contract is the same as a forward contract.

Answer: If

$$F_2 > F_1 e^{r(t_2 - t_1)}$$

an investor could make a riskless profit by

1. Taking a long position in a futures contract which matures at time t_1
2. Taking a short position in a futures contract which matures at time t_2

When the first futures contract matures, the asset is purchased for F_1 using funds borrowed at rate *r*. It is then held until time t_2 at which point it is exchanged for F_2 under the second contract. The costs of the funds borrowed and accumulated interest at time t_2 is $F_1 e^{r(t_2 - t_1)}$. A positive profit of

$$F_2 - F_1 e^{r(t_2 - t_1)}$$

is then realised at time t_2. This type of arbitrage opportunity cannot exist for long. Hence,

$$F_2 \le F_1 e^{r(t_2 - t_1)}.$$

Problem 5.19

Show that the growth rate in an index futures price equals the excess return of the portfolio underlying the index over the risk-free rate. Assume that the risk-free interest rate and the dividend yield are constant.

Answer: Suppose that F_0 is the futures price at time zero for a contract maturing at time T and F_1 is the futures price for the same contract at time t_1. It follows that

$$F_0 = S_0 e^{(r-q)T}$$

$$F_1 = S_1 e^{(r-q)(T-t_1)}$$

where S_0 and S_1 are the spot price at times zero and t_1, r is the risk-free rate, and q is the dividend yield. These equations imply that

$$\frac{F_1}{F_0} = \frac{S_1}{S_0} e^{-(r-q)t_1}$$

Define the excess return of the portfolio underlying the index over the risk-free rate as x. The total return is $r + x$ and the return realised in the form of capital gains is $r + x - q$. It follows that $S_1 = S_0 e^{(r+x-q)t_1}$ and the equation for F_1/F_0 reduces to

$$\frac{F_1}{F_0} = e^{xt_1}$$

which is the required result.

♣**Required Assignment Question 3 (Not in Hull, 8th ed)** **[8 points]**

In each case below provide a table (and plot) showing the relationship between profit/loss and final stock price. Ignore the impact of discounting.

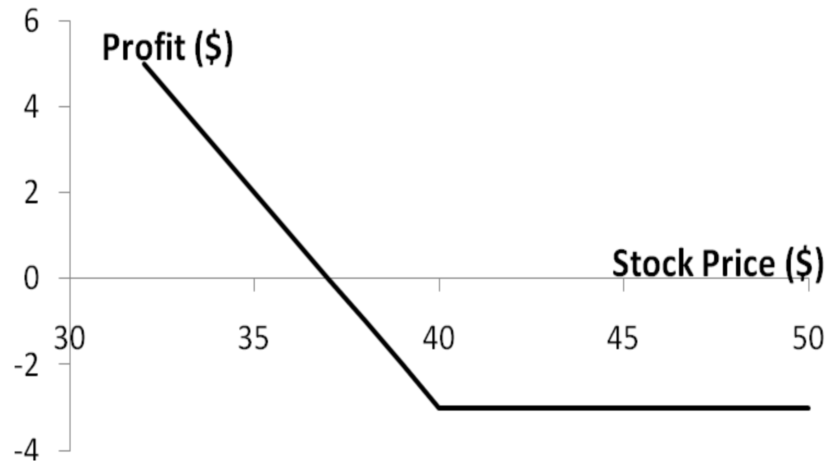
a. Call options with strike prices of $25 and $30 cost $7.90 and $4.18, respectively. Both have maturity of six months. Demonstrate the profit/loss pattern for a trading strategy (called bull spread) of buying the $25 call and selling the $30 call. Why would an investor be engaging such a strategy? **[**2.5 pts**]**
b. Put options with strike prices of $25, $30, and $35 cost $0.70, $2.14 and $4.57, respectively. All these options have maturity of one year. Demonstrate the profit/loss pattern for a trading strategy (called butterfly spread) of buying one $25 put option, buying one $35 put option and selling 2 $30 put option. Why would an investor be engaging such a strategy? **[**3 pts**]**
c. A six-month call option with a strike price of $35 costs $1.85. A six-month put option with a strike price of $25 costs $0.28. Demonstrate the profit/loss pattern for a trading strategy (called strangle) of buying the call and put options. Why would an investor be engaging such a strategy? **[**2.5 pts**]**

Answer: This question is an assigned problem for submission. Solution is in a separate sheet.

Problem 9.1

An investor buys a European put on a share for $3. The stock price is $42 and the strike price is $40. Under what circumstances does the investor make a profit? Under what circumstances will the option be exercised? Draw a diagram showing the variation of the investor's profit with the stock price at the maturity of the option.

Answer: The investor makes a profit if the price of the stock on the expiration date is less than $37. In these circumstances the gain from exercising the option is greater than $3. The option will be exercised if the stock price is less than $40 at the maturity of the option. The variation of the investor's profit with the stock price is depicted below.



Investor's profit pattern for Problem 9.1

Problem 9.12

A trader buys a call option with a strike price of $45 and a put option with a strike price of $40. Both options have the same maturity. The call costs $3 and the put costs $4. Draw a diagram showing the variation of the trader's profit with the asset price.

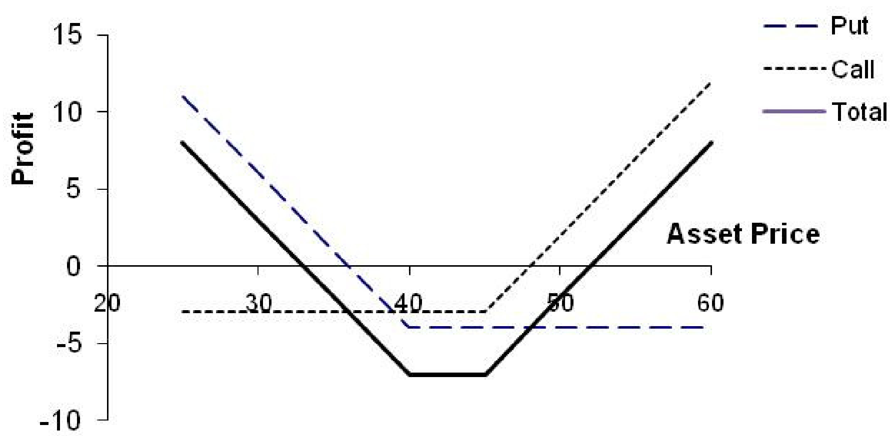
Answer: The figure below shows the variation of the trader's position with the asset price. We can divide the alternative asset prices into three ranges:

a) When the asset price less than $40, the put option provides a payoff of $40-S_T$ and the call option provides no payoff. The options cost $7 and so the total profit is $33-S_T$.

b) When the asset price is between $40 and $45, neither option provides a payoff. There is a net loss of $7.

c) When the asset price greater than $45, the call option provides a payoff of S_T-45 and the put option provides no payoff. Taking into account the $7 cost of the options, the total profit is S_T-52.

The trader makes a profit (ignoring the time value of money) if the stock price is less than $33 or greater than $52. This type of trading strategy is employed by a trader who speculates that in the future the stock price will have a large movement but unsure which direction this will be.



Profit pattern for the trading strategy in Problem 9.12

♣Required Assignment Question 4 (Cvitanic and Zapatero, problem 18 of chapter 1) [4 points]

At time zero you enter into a short position in futures contract on 20 shares of stock XYZ at the futures price of $50.00. Moreover, you write (sell) 5 "exotic" options of the following type: they are put options, but using as the underlying asset the average of today's stock price and the stock price at maturity of the underlying asset. The option's price is $52, the option selling price today is $5.00 per option and today's stock price is $49.00 per share. The maturity of all your positions is T = months. What is your total profit or loss two months from now if

(a) at maturity the price of one stock share is $57.00? **[**2 pts**]**

(b) at maturity the price of one stock share is $47.00? **[**2 pts**]**

Answer: This question is an assigned problem for submission. Solution is in a separate sheet.

Problem 9.13

Explain why an American option is always worth at least as much as a European option on the same asset with the same strike price and exercise date.

Answer: The holder of an American option has all the same rights as the holder of a European option and more. It must therefore be worth at least as much. If it were not, an arbitrageur could short the European option and take a long position in the American option.

Problem 9.14

Explain why an American option is always worth at least as much as its intrinsic value.

Answer: The holder of an American option has the right to exercise it immediately. The American option must therefore be worth at least as much as its intrinsic value. If it were not an arbitrageur could lock in a sure profit by buying the option and exercising it immediately.

Problem 9.15

Explain carefully the difference between writing a put option and buying a call option.

Answer: Writing a put gives a payoff of $\min(S_T - K, 0)$. Buying a call gives a payoff of $\max(S_T - K, 0)$. In both cases the potential payoff is $S_T - K$. The difference is that for a written put the counterparty chooses whether you get the payoff (and will allow you to get it only when it is negative to you). For a long call you decide whether you get the payoff (and you choose to get it when it is positive to you.)

Problem 10.1

List the six factors affecting option prices.

Answer: The six factors affecting stock option prices are the stock price, strike price, risk-free interest rate, volatility, time to maturity, and dividends.

Problem 10.4

Give two reasons that the early exercise of an American call option on a non-dividend paying stock is not optimal. The first reason should involve the time value of money. The second reason should apply even if interest rates are zero.

Answer: Delaying exercise delays the payment of the strike price. This means that the option holder is able to earn interest on the strike price for a longer period of time. Delaying exercise also provides insurance against the stock price rising above the strike price (i.e., enables the holder to buy at a low price when the stock price is high in the market) by the expiration date. Assume that the option holder has an amount of cash K and that interest rates are zero. When the option is exercised early it is worth S_T at expiration. Delaying exercise means that it will be worth $\max(K, S_T)$ at expiration.

Problem 10.5

"The early exercise of an American put is a trade-off between the time value of money and the insurance value of a put". Explain this statement.

Answer: An American put when held in conjunction with the underlying stock provides insurance. It guarantees that the stock can be sold for the strike price, *K*. If the put is exercised early, the insurance ceases. However, the option holder receives the strike price immediately and is able to earn interest on it between the time of the early exercise and the expiration date.

Problem 10.8-8th

Explain why the arguments leading to put–call parity for European options cannot be used to give a similar result for American options.

Answer: When early exercise is not possible, we can argue that two portfolios that are worth the same at time T must be worth the same at earlier times. When early exercise is possible, the argument falls down. Suppose that $P+S>C+Ke^{-rT}$. This situation does not lead to an arbitrage opportunity. If we buy the call, short the put, and short the stock, we cannot be sure of the result because we do not know when the put will be exercised.

Problem 10.10

What is a lower bound for the price of a two-month European put option on a non-dividend-paying stock when the stock price is \$58, the strike price is \$65, and the risk-free interest rate is 5% per annum?

Answer: The lower bound is $65e^{-0.05\times 2/12}-58=\$6.46$

Problem 10.11

A four-month European call option on a dividend-paying stock is currently selling for \$5. The stock price is \$64, the strike price is \$60, and a dividend of \$0.80 is expected in one month. The risk-free interest rate is 12% per annum for all maturities. What opportunities are there for an arbitrageur?

Answer: The present value of the strike price is $60e^{-0.12\times 4/12}=\$57.65$. The present value of the dividend is $0.80e^{-0.12\times 1/12}=0.79$. Since

$$5<64-57.65-0.79$$

the condition $c\geq \max(S_0-D-Ke^{-rT},0)$ is violated. An arbitrageur should buy the option and short the stock. This generates $64-5=\$59$. The arbitrageur invests \$0.79 of this at 12% for one month to pay the dividend of \$0.80 in one month. The remaining \$58.21 is invested for four months at 12%. Regardless of what happens a profit will materialise.

If the stock price declines below \$60 in four months, the arbitrageur loses the \$5 spent on the option but gains on the short position. The arbitrageur shorts when the stock price is \$64, has to pay dividends with a present value of \$0.79, and closes out the short position when the stock price is \$60 or less. Since \$57.65 is the present value of \$60, the short position generates at least $64-57.65-0.79=\$5.56$ in present value terms. The present value of the arbitrageur's gain is therefore at least $5.56-5.00=\$0.56$.

If the stock price is above \$60 at the expiration of the option, the option is exercised. The arbitrageur buys the stock for \$60 in four months and closes out the short position. The present value of the \$60 paid for the stock is \$57.65 and as before the dividend has a present value of \$0.79. The gain from the short position and the exercise of the option is therefore exactly $64-57.65-0.79=\$5.56$. The arbitrageur's gain in present value terms is exactly $5.56-5.00=\$0.56$.

Problem 10.14

The price of a European call that expires in six months and has a strike price of \$30 is \$2. The underlying stock price is \$29, and a dividend of \$0.50 is expected in two months and again in five months. The term structure is flat, with all risk-free interest rates being 10%. What is the price of a European put option that expires in six months and has a strike price of \$30?

Answer: Using the put-call parity

$$c + Ke^{-rT} + D = p + S_0$$

we have

$$p = c + Ke^{-rT} + D - S_0$$

In this case

$$p = 2 + 30e^{-0.1\times 6/12} + (0.5e^{-0.1\times 2/12} + 0.5e^{-0.1\times 5/12}) - 29 = 2.51$$

In other words the put price is \$2.51.

Problem 10.15

Explain carefully the arbitrage opportunities in Problem 10.14 if the European put price is \$3.

Answer: If the put price is \$3.00, it is too high relative to the call price. An arbitrageur should buy the call, short the put and short the stock. This generates $-2+3+29=\$30$ in cash which is invested at 10%. Regardless of what happens a profit with a present value of $3.00-2.51=\$0.49$ is locked in.

If the stock price is above \$30 in six months, the call option is exercised, and the put option expires worthless. The call option enables the stock to be bought for \$30, or $30e^{-0.10\times 6/12}=\$28.54$ in present value terms. The dividends on the short position cost $0.5e^{-0.1\times 2/12}+0.5e^{-0.1\times 5/12}=\$0.97$ in present value terms so that there is a profit with a present value of $30-28.54-0.97=\$0.49$.

If the stock price is below \$30 in six months, the put option is exercised and the call option expires worthless. The short put option leads to the stock being bought for \$30, or $30e^{-0.10\times 6/12}=\$28.54$ in present value terms. The dividends on the short position cost $0.5e^{-0.1\times 2/12}+0.5e^{-0.1\times 5/12}=\$0.97$ in present value terms so that there is a profit with a present value of $30-28.54-0.97=\$0.49$.

Problem 10.16

The price of an American call on a non-dividend-paying stock is \$4. The stock price is \$31, the strike price is \$30, and the expiration date is in three months. The risk-free interest rate is 8%. Derive upper and lower bounds for the price of an American put on the same stock with the same strike price and expiration date.

Answer: From equation (10.7)

$$S_0 - K \le C - P \le S_0 - Ke^{-rT}$$

In this case

$$31 - 30 \le 4 - P \le 31 - 30e^{-0.08\times 0.25}$$

or

$$1.00 \le 4.00 - P \le 1.59$$

or

$$2.41 \le P \le 3.00$$

Upper and lower bounds for the price of an American put are therefore \$2.41 and \$3.00.

Problem 10.17 Explain carefully the arbitrage opportunities in Problem 10.16 if the American put price is greater than the calculated upper bound.

Answer: If the American put price is greater than \$3.00 an arbitrageur can sell the American put, short the stock, and buy the American call. This realises at least $3 + 31 - 4 = \$30$ which can be invested at the risk-free interest rate. At some stage during the 3-month period either the American put or the American call will be exercised. The arbitrageur then pays \$30, receives the stock and closes out the short position. The cash flows to the arbitrageur are $+\$30$ at time zero and $-\$30$ at some future time. These cash flows have a positive present value.

Problem 10.19 Prove the result in equation (10.11). (Hint: For the first part of the relationship consider (a) a portfolio consisting of a European call plus an amount of cash equal to $D + K$ and (b) a portfolio consisting of an American put option plus one share.)

Answer: Let c and p denote the respective European call and put option prices, and C and P denote the respective American call and put option prices. The present value of the dividends will be denoted by D. As shown in the class lecture, when there are no dividends

$$C - P \le S_0 - Ke^{-rT}$$

Dividends reduce C and increase P. Hence this relationship must also be true when there are dividends.

For a further relationship between C and P, consider

Portfolio I: one European call option plus an amount of cash equal to $D + K$

Portfolio J: one American put option plus one share

Both options have the same exercise price and expiration date. Assume that the cash in portfolio I is invested at the risk-free interest rate. If the put option is not exercised early, portfolio J is worth

$$\max(S_T, K) + De^{rT}$$

at time T. Portfolio I is worth

$$\max(S_T - K, 0) + (D + K)e^{rT} = \max(S_T, K) + De^{rT} + Ke^{rT} - K$$

at this time. Portfolio I is therefore worth more than portfolio J. Suppose next that the put option in portfolio J is exercised early, say, at time τ. This means that portfolio J is worth at most $K + De^{r\tau}$ at time τ. However, even if the call option were worthless, portfolio I would be worth $(D + K)e^{r\tau}$ at time τ. It follows that portfolio I is worth more than portfolio J in all circumstances. Hence

$$c + D + K \ge P + S_0$$

Since $C \ge c$,

$$C - P \ge S_0 - D - K$$