Maths 190

Assignment No. 2

Due Date: 09:00HRS, 08 March 2017

Venue: Computer Lab, 1st Floor, CAS Bldg

N.B. ***Your assignment paper must include the Marking Scheme sheet as a cover page. This can be downloaded from the course website. Failure to follow this instruction will result to a 2-point deduction on your assignment mark. Additionally, each instruction (if any) not correctly followed in the marking scheme sheet will also lead to a 0.2-point deduction.***

NOTE: Only problems marked with ♣ are required for submission; there are 4 of them and they are found on pages 3 and 4. The other problems are intended as additional practice exercises.

GUIDELINES ON SUBMITTING ASSIGNMENTS

- Do not submit your rough work! Do the problem set and then re-write it at least once - neatly, with adequate amount of clear explanation. The rewriting stage is the most important one for finding errors in one's work, and it will also deepen your understanding of the subject matter. Assignments are marked for both technical correctness and elegance of presentation.

- Bear in mind to include a sufficient amount of explanation about your work so that any marker does not have to guess what you mean. The grader of your work will determine if you understand what you are writing, not merely that you reach the particular correct answer.

- On questions where a computer output is required, include the output in the text of your answer at the appropriate locations - do not put it all in a bunch at the end of your assignment. Unless, you are instructed to submit your work electronically or via a USB, you are expected to hand in a PRINTED COPY. ***Assignments sent via e-mail will not be accepted.***

- YOU MUST WRITE YOUR OWN WORK IN YOUR OWN WORDS, using full sentences and proper English grammar. It is your responsibility to familiarise yourself with the provisions of the University Regulation concerning academic

integrity and honesty. ***Any behaviour that can potentially lead to plagiarism and cheating is a serious offence and carries with it severe penalty.***

For problems not required for submission, you will learn and benefit more if you attempt solving them first before looking at their solutions.

BINOMIAL OPTION PRICING PROBLEMS

Problem 12.2
Explain the no-arbitrage and risk-neutral valuation approaches to valuing a European option using a one-step binomial tree.

Problem 12.3
What is meant by the delta of a stock option?

Problem 12.5
A stock price is currently $100. Over each of the next two six-month periods it is expected to go up by 10% or down by 10%. The risk-free interest rate is 8% per annum with continuous compounding. What is the value of a one-year European call option with a strike price of $100?

Problem 12.6
For the situation considered in Problem 12.5, what is the value of a one-year European put option with a strike price of $100? Verify that the European call and European put prices satisfy put–call parity.

Problem 12.8
Consider the situation in which stock price movements during the life of a European option are governed by a two-step binomial tree. Explain why it is not possible to set up a position in the stock and the option that remains riskless for the whole of the life of the option.

Problem 12.10
A stock price is currently $80. It is known that at the end of four months it will be either $75 or $85. The risk-free interest rate is 5% per annum with continuous compounding. What is the value of a four-month European put option with a strike price of $80? Use no-arbitrage arguments.

Problem 12.11
A stock price is currently \$40. It is known that at the end of three months it will be either \$45 or \$35. The risk-free rate of interest with quarterly compounding is 8% per annum. Calculate the value of a three-month European put option on the stock with an exercise price of \$40. Verify that no-arbitrage arguments and risk-neutral valuation arguments give the same answers.

Problem 12.14
A stock price is currently \$25. It is known that at the end of two months it will be either \$23 or \$27. The risk-free interest rate is 10% per annum with continuous compounding. Suppose S_T is the stock price at the end of two months. What is the value of a derivative that pays off S_T^2 at this time?

Problem 12.15
Calculate u, d, and p when a binomial tree is constructed to value an option on a foreign currency. The tree step size is one month, the domestic interest rate is 5% per annum, the foreign interest rate is 8% per annum, and the volatility is 12% per annum.

♣REQUIRED PROBLEM #1 [4 points]
Additional Problem 1
A stock price is currently \$25. It is known that at the end of 4 months it will be either \$30 or \$21. The risk-free rate of interest with continuous compounding is 12% per annum. Calculate the value of a 4-month European call option with an exercise price of \$24. Verify that no-arbitrage arguments and risk-neutral valuation arguments give the same answer.

♣REQUIRED PROBLEM #2 [5 points]
Additional Problem 2
In a two-period binomial model with $r=1\%$ per period, the current stock price is \$100, and $u=1.02$ and $d=0.98$. Consider an option that expires after two periods, and pays the value of the squared stock price, $S(t)^2$, if the stock price $S(t)$ is higher than \$100 when the option is exercised. Otherwise (when $S(t)$ is less than or equal to \$100), the option pays zero. If the option under consideration is an American-type, find its price.

BASIC ELEMENTS OF STOCHASTIC PROCESSES

♣Required Assignment Problem #3 [7 points]

Consider a 2-step trinomial non-recombining lattice tree model. For each step, there are three possibilities for the stock price: an up movement (u), a down movement (d) or no movement (n).

(a) Write down the set or sample space, Ω, containing all possible outcomes for this 2-step trinomial tree. If we consider the collection of all subsets of Ω, how many subsets are there in this collection? [1 point]

(b) Suppose the respective probabilities of events $\{u\}$ and $\{d\}$ are 3/7 and 2/7. Define or construct the probability measure for each individual element $\omega \in \Omega$. [1.5 points]

(c) Write down the σ-algebra or σ-field $\{\Im_i\}$ keeping track the outcomes for each time step $i = 0, 1, 2$. [1.5 points]

(d) Consider the Ω given in (a). Under the trinomial asset pricing model suppose $S_0 = 50$, $d = 10/11$ and $u = 12/11$; clearly, $\{S_i\}$ is a stochastic process, i.e., S_i's are random variables for $i = 0, 1, 2$. Find $S_1(\varpi)$, i.e., what is the function $S_1(\varpi)$? [2 pts]

(e) Consider the interval $B=[2\pi^e-1, 2e^\pi+1]$. What is $S_1^{-1}(B)$? Recall that S_1 is a random variable and by definition it maps Ω into $\Re$. [1 pt]

♣Required Assignment Problem #4 [4 points]

Assume a single-period model with $r = 0.05$ and S(0) =100. After one period, the stock price can either go up to 101 or down to 99.

(a) Find a replicating portfolio for a put option with the pay off function $g(s) = (100\text{-}s)^+ = \max(100\text{-}s, 0)$. **Use a replicating portfolio consisting of α amount held in a savings account and Δ shares of holdings in stock.** [3 pts] [**Note:** *You will get a zero for this question if you construct a different replicating portfolio.*]

(b) Using your replicating portfolio in (a), what is the price of the put option in question? [1 pt]

Additional Problem (Not for submission)

Consider a single-period CRR model with interest rate 0.05, $S(0)=10$, $u=1.2$ and $d=0.98$. Suppose you have written an option that pays the value of the square root of the absolute value of the difference between the stock price at maturity and \$10.00; that is, it pays $\sqrt{|S(1)-10|}$. How many shares of the stock should you buy to replicate this pay-off? What is the cost of the replicating portfolio?