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Division of Physical Sciences & Mathematics
University of the Philippines in the Visayas
Miag-ao, Iloilo, Philippines

Maths 190
Mathematical Models in Finance
2nd Semester (Fast-tracked delivery: Jan-Mar 2017)

Course Instructor: Dr Rogemar S Mamon
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Course Communication: Announcements and certain course materials will be made available via a password-protected course website. If internet connection or the server has an issue, course materials will be sent via email. Please ensure that you provide your email address in the Attendance Sheet.

Check the course website and your email before each meeting.

Office Hours: 13:15– 14:15 Mon, Wed and Thu at Room B10. A heads up will be much appreciated. Please send a brief email stating your intent, nature of your queries and the time you will be dropping by.

Lectures: 14:30-16:00 Monday and Thursday; 09:00-11:00 Wednesday. All lectures will be held in Room B6.

Prerequisites: Maths 55, Statistics to the level of Stats 110/111 (and enthusiasm to learn quantitative finance and willingness to work harder, *given the fast-tracked nature of the course*).

If you do not have the above prerequisites, you may enroll by **special permission.**

Remark: This course covers the convention in the financial markets, building blocks of financial instruments, and the basic quantitative tools in pricing and risk management of contracts. **This course employs a probabilistic approach.** *An alternative (which complements our methodology) is the* ***PDE approach.***

Description: Forwards, futures, swaps, market conventions, option pricing and replication, American options, properties of stock option prices, binomial model. Continuous-time models, Brownian motion, stochastic integrals, Ito's lemma. Black-Scholes-Merton market model, arbitrage and market completeness, Black-Scholes PDE, risk-neutral pricing and martingale measures. Greeks and hedging, extensions of Black-Scholes model, implied volatility. Value-at-risk. Simulation.

Main Text: John Hull (2012). *Options, Futures, and Other Derivatives*, 8th edition. Pearson Prentice Hall. **[JH]**

Reference: Jaksa Cvitanic and Fernando Zapatero (2004). *Introduction to the Economics and Mathematics of Financial Markets*. MIT Press, Cambridge, Massachusetts. **[JCFZ]**

E-copies of textbooks are available from the instructor.
Weekly summary of lectures will be provided by the instructor via email.

Long exams: Two 2-hour tests will be given in class on **22 February 2017** (Wednesday) and **15 March 2017** (Wednesday). Venue: TBA
Time: 09:00-11:00

Evaluation:

Students will be assessed on the basis of 2 assignments (15%); 1 group project (10%); 2 midterms (45%) and a 3-hour sit-in final examination (30%).

G1 = 0.15*A + 0.10*P + 0.45*M + 0.30*F

G2 = 0.15*A + 0.10*P + 0.30*M + 0.45*F if F>M

G3 = 100% Final exam score if all other components > 60

Final Grade = max(G1, G2, G3), where A=Assignments, P=project, M=midterm, and F=final exam

Assignments and project presentation:

There will be 2 assignments in this course one month apart. Each assignment will be sent via email one week before the due date. These due dates are: ***09 February 2017*** for Assignment #1, ***08 March 2017*** for Assignment #2, and ***23 March 2017*** for Assignment #3. The project presentation will be in the week of 20-24 March. **Submit your assignments before the lecture begins on the due date. Under no circumstances will late assignments be accepted.**

Final Exam: 09:00-12:00, Saturday, 01 April 2017

Missed Assignments and Midterm tests:	**<u>There will be no make-up for missed assignments and long exams</u>.** For those who are unable to hand in an assignment or write an exam, the standard practice will be that the weight of the missed course requirement will be transferred to the final exam. Supporting documentation must be provided to the instructor.

VERY IMPORTANT:

Academia seeks to provide an environment of free and creative enquiry within which critical thinking, humane values, and practical skills are cultivated and sustained. A first step in creating and maintaining this environment in the class is to have some ***mutual expectations*** between the instructor and the students on the atmosphere for instruction in the course.

The instructor expects:

- ***that students will not engage in disruptive behaviour, including talking out of turn or social chatting, during class time;***
- ***that students come to class on time;***
- that the submitted version of an assignment represents the student's independent work—this does not preclude true collaboration in discussing problems (the submission of a fraudulent assignment is a **scholastic offence**, and will be dealt with accordingly).

Students could expect:

- to receive from the instructor, at the beginning of term, a clear course outline;
- that the instructor will come prepared for class and attend lectures regularly;
- that the instructor will begin punctually and not run beyond the allotted time for the class;
- that out of class the instructor will be available for scheduled appointments at posted office hours; and
- that marked long exams and assignments will be returned within a reasonable period of time after submission.

Maths 190 *[Mathematical Models in Finance]*
(Second Semester, Jan-Mar 2017)
Topics to be covered with relevant readings

❖ Brief Review of
(i) derivatives and how they are used by different types of traders. **Relevant readings:** Chapter 1 of JH; sections 1.3 and 1.4 of JCFZ.
(ii) Determination of forward and futures prices (To demonstrate no-arbitrage pricing principle). **Relevant readings:** Chapter 5 of JH; section 6.2 of JCFZ

❖ Time value of money (discrete to continuous compounding/discounting). Bond pricing. **Relevant readings:** Sections 4.1-4.4 of JH.

❖ Properties of stock option prices. **Relevant readings:** Chapter 10 of JH.

❖ Discrete-time market models, option pricing by replication, risk-neutral valuation and martingale measures, and the fundamental theorem of asset pricing. Discrete-time Black-Scholes pricing framework (the Cox-Ross-Rubinstein binomial option pricing model). **Relevant readings:** Chapter 12 of JH; sections 3.1-3.2, 6.3.1-6.3.5 of JCFZ.

❖ Elements of stochastic processes in discrete time. *This topic is based on instructor's notes.*

❖ Brownian motion, Itô's lemma, risk-neutral/martingale measure, Black-Scholes PDE, risk-neutral valuation and the connection between pricing PDE and conditional expectation under the martingale measure. **Relevant readings:** Chapter 13 and sections 14.1-14.8 in JH; section 7.2 in JCFZ

❖ Introduction to the basics of Monte-Carlo methods in finance. Simulation of correlated assets. **Relevant readings:** Sections 13.3, 20.6 in JH; section 11.2.1 in JCFZ

❖ The "Greeks" and hedging in practice. **Relevant readings:** Chapter 18 in JH; sections 9.3.1-9.3.5, 9.3.7, 9.3.8 in JCFZ

❖Extension of Black-Scholes to the pricing of options on stock indices, currencies and futures. **Relevant readings:** Sections Chapter 16 in JH; sections 7.4.2, 7.5.1, 7.5.2 in JCFZ

❖ Value-at-risk. **Relevant readings:** Chapter 21 of JH; section 5.2 of JCFZ.

❖ Volatility smiles/smirks, skews and volatility surfaces. **Relevant readings:** Chapter 19 in JH. No relevant materials from JCFZ.

❖ Swaps. **Relevant readings:** Chapter 7 of JH.

❖ Time-varying volatility and correlation models. **Relevant readings:** GARCH/EWMA, Chapter 22 of JH.

The End of Course Outline

Purpose of Math'l Methods in Finance

- Intro to important concepts in pricing derivative securities
- Provide quantitative tools to quantify market risk
- Valuation and risk management of financial instruments
- Prepare students for the necessary concepts needed for the treatment of asset pricing in the graduate level
- Give an indication of (career) opportunities in the financial industry and other related fields that heavily make use of mathematical-finance approaches

The nature of this course

- Is this course challenging?

It depends!

Your main focus should be on:

Intuition behind the "relatively simple" mathematics, understanding model assumptions, learning recent innovations and fundamental theories that form the underpinnings in financial valuation and risk management. Elements of stochastic processes in discrete-time will be introduced.

The course will get mathematical when we reach continuous-time models.

***Start reading JH* on:**

- The role of derivatives: why do they exist?
- How are derivatives employed by arbitrageurs, hedgers and speculators?
- How does one determine prices for forwards and futures?

Developments with the decade

Foreign exchange

- Freeing of exchange and capital controls in the past 3 decades (more FX trades)
- Need to eliminate FX risk

Interest rate

- Interest rate controls were eliminated and coincided with increases in government budget deficits
- Large new issues of government debts in almost all industrialised nations

So, what contributed mainly to the prolific increase in markets for derivative products?

How was the need to hedge or control the risk associated with FX and interest rate uncertainty addressed?

It was addressed by the market partly through ….

- Conceptual understanding of the ***structure, functioning and pricing*** of these derivative products is paramount.

- Theoretical valuation models are directly applicable to these new products.

- Important question: *How does a financial institution (FI) "price correctly" a financial instrument that is not even trading yet in the market?*

Introduction of new products become easier and less costly.

- New exchanges and market places came into being in response to the growth of derivative securities.

- Deregulation of financial services in the 1980's.

Theoretical developments covered in the pricing of derivatives

- ARBITRAGE THEOREM
- BLACK-SCHOLES MODEL (Black & Scholes, 1973). We shall consider the discrete-time version **[Cox-Ross-Rubinstein (CRR), 1979]**. Results published in the Journal of Financial Economics (JFE)
- METHODOLOGY OF EQUIVALENT MARTINGALE MEASURE

Important concepts to be emphasised in derivative valuation

- No-arbitrage pricing
- Risk-neutral probabilities
- Martingales

Concepts, ideas and theories in this course formed the very foundation of finance

Evidence: Ideas and theories recognised by the Nobel Memorial Prize for contributions in Economic Sciences

- Method to determine the price of derivatives (Scholes and Merton, 1997)
- Portfolio theory and diversification (Markowitz, 1990)
- Investment performance analysis (Sharpe, 1990)

Review of derivatives

Practitioner's definition: Derivative securities are financial contracts that "derive" their value from cash market instruments such as stocks, bonds, currencies and commodities.

Academic definition: A financial contract is a derivative security, or a contingent claim, if its value at expiration date T is determined exactly by the market price of the underlying cash instrument at time T.

Caveat: The underlying financial variable does not necessarily have to be asset price.

Financial contracts are written on interest rates, inflation, temperature at a particular pt of a city, level of snow of a certain ski resort, or even the performance of a hockey team.

Note that since movement of financial variables, economic indicators, weather-related measurements or winning of sports are full of uncertainty. **This is where knowledge of probability and stochastic processes will come into the picture.**

Give examples of a ***financial derivative*** that you have so far studied, read or heard. Why are considered derivatives?

Notation:

F:=price of a derivative security

$F(S_T)$:=value of derivative at time T; S is the underlying asset and T is the maturity date

$F(S_t, t)$: = price of a derivative product written on the underlying asset S_t at time t.

Types of derivatives

Three general classification

- Futures and forwards
- Options
- Swaps

Basic building blocks: (i) forwards and (ii) options

Others are hybrid securities.

Main Groups of Underlying Asset/Variables

- ***Stocks***
- ***Currencies***
- ***Interests***
- ***Indices***
- ***Commodities***