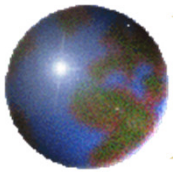
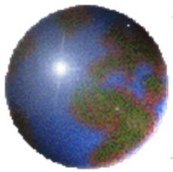


AGENDA OF THE DAY

- Recap of financial contracts and valuation
- Energy markets and weather derivatives
- Examples of applications of derivatives and option-pricing approach
- *Continuation of the Binomial option pricing approach* (From discrete-time setting to continuous-time setting – the ***Black-Scholes-Merton Model***)



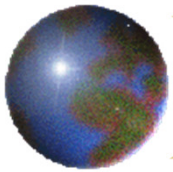
RECAP OF FINANCIAL CONTRACTS & VALUATION

- A **(financial) derivative** or a **contingent claim** is a financial instrument whose value depends on (*or derives from*) the values of other, more basic, underlying variables.

- **Examples of underlying variables/prices**: ***bonds*** (depend on interest rates); ***stock options*** (depend on stock prices); ***pension benefits*** (interest rates, survival probability, various financial variables); ***insurance policy,*** etc.
- *Derivatives could depend on almost any variable, from the price of hogs to the amount of snow falling at a certain ski resort.*
- There is now **active trading** in **financial** derivatives **electricity** derivatives, **weather** derivatives, and **insurance** derivatives.

RATIONALE of how derivatives came into being

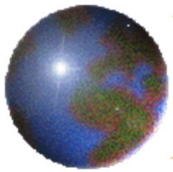
SECURITIES CLASSIFICATION MATRIX



TYPES OF CONTRACTS

- A **forward contract** is an agreement to buy or sell an asset at a certain future time for a certain price. **Futures contracts** are similar to forwards but *traded on an exchange.*

- A **swap** is an over-the-counter (OTC) agreement between two companies to exchange cash flows in the future; & could be analysed as extension of forwards.



TYPES OF CONTRACTS (CONTINUED)

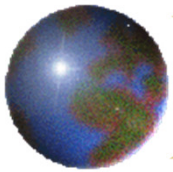
An **option** gives the **holder** the right (**but not the obligation**) to buy or sell an asset at a certain date (contract's maturity date T) for a certain price (strike price X).

call option - right to buy an asset at agreed price X so that pay-off = $\max(S_T - X, 0)$

put option - right to sell an asset at agreed price X so that pay-off=$\max(X - S_T, 0)$

Note: Options can be **European** or **American**. European options can only be exercised at time T whilst American options can be exercised prior to T.

Holder of a contract is called the *buyer* (and has the right); the issuer of the contract is called the *write/seller* (obligated to deliver or acquire the asset if the option is exercised).

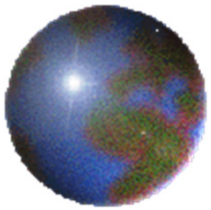


Motivation

- Forward contracts are designed to ***neutralise risk*** by fixing the price.
- ***Options provide insurance.*** They offer a way for investors to protect themselves against adverse price movements in the future whilst still allowing them to benefit from favourable price movements.
- **Example** - Portfolio insurance. *Option to sell 100,000 shares of* IBM *stock* for $100/share. Buy put option on 1,000 contracts. **Guaranteed $10M** if stock price goes below $100. *DON'T HAVE TO EXERCISE CONTRACT WHEN PRICE IS ABOVE $100.*
- Options require an up-front fee (**premium)**.

Interactive summary of derivatives concepts AND further extensions

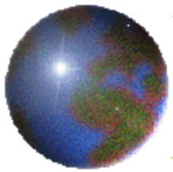
- Video show – stay tuned!



Energy and Commodity Derivatives

Again, underlying variables ***are not limited to financial asset*** prices or economic indicators!

SLIDE SHOW – stay tuned!



APPLICATIONS OF DERIVATIVES TO COMMODITY AND ENERGYMARKETS

I. *Agricultural Commodities*

- Corn, wheat, soybeans, cocoa, coffee, sugar, cotton, frozen orange juice, cattle, hogs, pork bellies, etc

CHARACTERISTICS

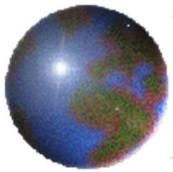
- Supply-demand measured by stocks-to-use ratio
- **Seasonality** and **mean reversion** in prices (farmers have a choice about what they produce)
- ***Weather important***

II. *Metals*

- Gold, silver, platinum, palladium, copper, tin, lead, zinc, nickel, aluminium, etc

CHARACTERISTICS

- No seasonality; weather unimportant
- ***Investment*** vs ***consumption*** metals
- Some mean reversion (it can become uneconomic to extract a metal)
- Recycling (source estimate: 10%)



III. *Energy commodities*

- **Main energy sources**
  - Oil
  - Gas
  - Electricity
- ***All have mean-reverting prices***
- Gas and electricity prices ***exhibit jumps***

IV. *Crude Oil*

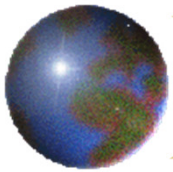
- Largest commodity market in the world

CHARACTERISTICS

- Many grades. For example
  - Brent crude oil (sourced out from the North Sea)
  - West Texas Intermediate (WTI) crude
- Refined products, for example:
  - Gasoline
  - Heating oil
  - Kerosene
  - **Jet/aviation fuel** (unleaded **kerosene** for **Jet** A-1 or **naphtha-kerosene** blend for **Jet** B)

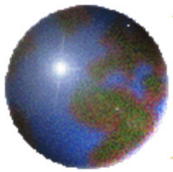
V. *Natural Gas and Electricity*

- Deregulated
- Elimination of government monopolies
- Producer and supplier not necessarily the same



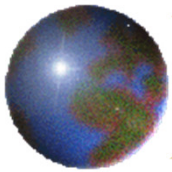
OIL DERIVATIVES

- Virtually all derivatives available on stocks and stock indices are also available in the OTC market ***with oil as the underlying asset.***
- Futures and futures options traded on the New York Mercantile Exchange (NYMEX) and the International Petroleum Exchange (IPE) are also popular.



NATURAL GAS DERIVATIVES

- A typical OTC contract is for the delivery of a specified amount of natural gas at a roughly uniform rate to specified location during a month.
- NYMEX and IPE trade contracts that require delivery of 10,000 million British thermal units (BTUs) of natural gas to a specified location



ELECTRICITY DERIVATIVES

- Electricity is an unusual commodity ***in that it cannot be stored.***

- The U.S is divided into about 140 control areas and a market for electricity is created by trading between control areas.

ELECTRICITY DERIVATIVES

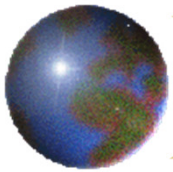
(*continued*)

Aiding SUSTAINABILITY OF ENERGY SUPPLY IN NORTH AMERICA & EUROPE

- A typical contract allows one side **to receive a specified number of megawatt hours** for a specified price at a specified location during a particular month

- Types of contracts:

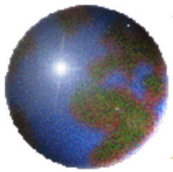
  5x8, 5x16, 7x24, **daily** or **monthly** exercise



Commodity prices

- EVOLUTION OF futures prices can be used to define the process followed by a commodity price in a risk-neutral world.

- We can ***build in mean reversion and use a process for constructing trinomial trees*** that is analogous to that used for interest rates.



The Process for the Commodity Price

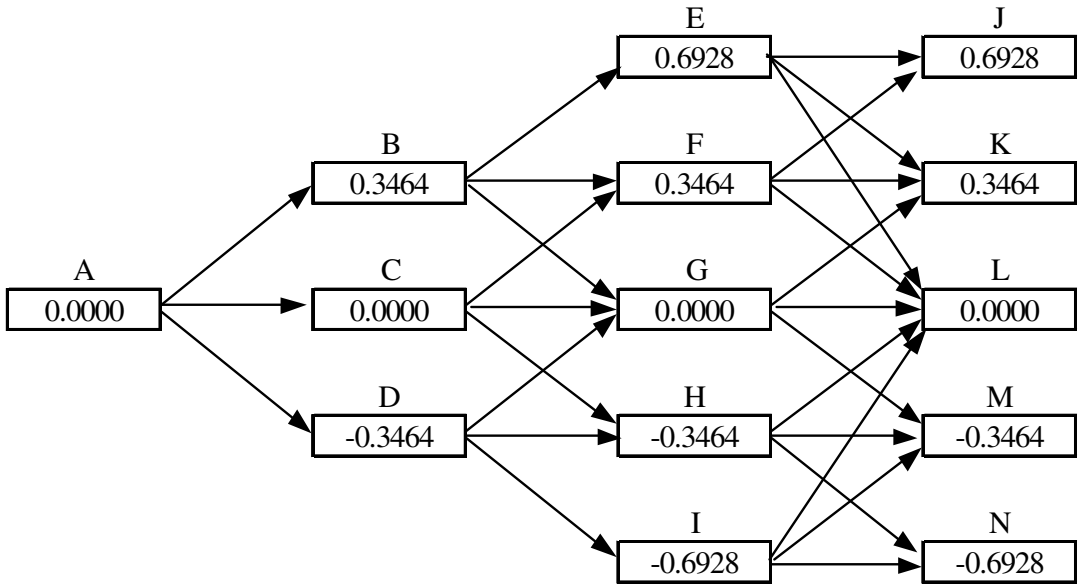
A simple ***mean-reverting*** process is

$$d\ln(S_t) = [\theta(t) - a\ln(S_t)]\,dt + \sigma\,dW_t$$

WE NEED TO DEVELOP **SIMPLE BUT ROBUST** ***MATHEMATICAL MODELS FOR THE EVOLUTION OF VARIOUS UNDERLYING VARIABLES!***

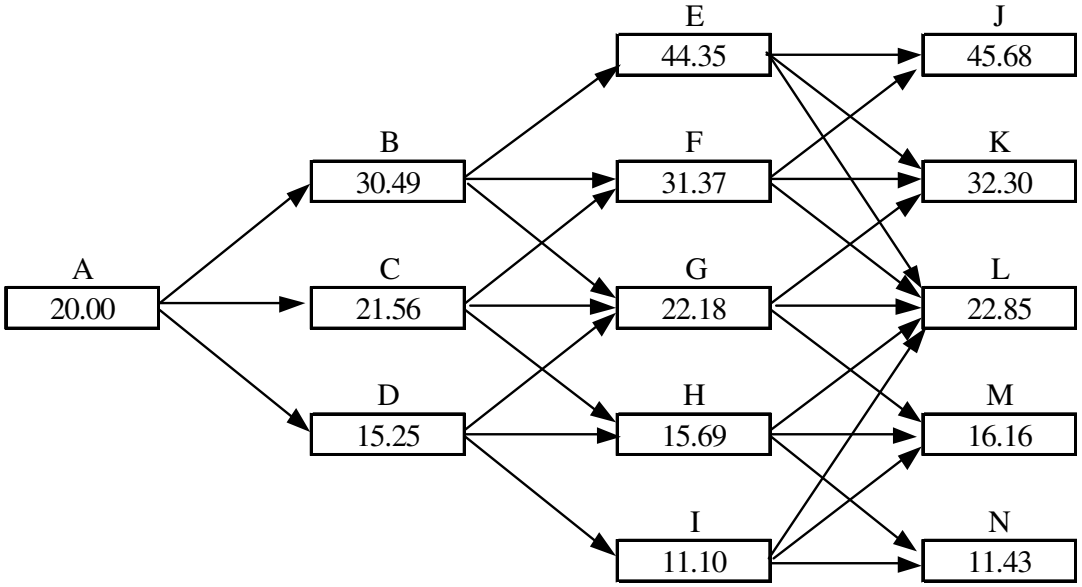
From Binomial to Trinomial Tree **(when there is mean reversion)**

Trinomial tree for ln S_t Assuming $\theta(t)=0$

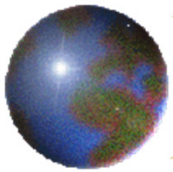


Node	A	B	C	D	E	F	G	H	I
p_u	0.1667	0.1217	0.1667	0.2217	0.8867	0.1217	0.1667	0.2217	0.0867
p_m	0.6666	0.6566	0.6666	0.6566	0.0266	0.6566	0.6666	0.6566	0.0266
p_d	0.1667	0.2217	0.1667	0.1217	0.0867	0.2217	0.1667	0.1217	0.8867

Resulting trinomial tree for a price level



Node	A	B	C	D	E	F	G	H	I
p_u	0.1667	0.1217	0.1667	0.2217	0.8867	0.1217	0.1667	0.2217	0.0867
p_m	0.6666	0.6566	0.6666	0.6566	0.0266	0.6566	0.6666	0.6566	0.0266
p_d	0.1667	0.2217	0.1667	0.1217	0.0867	0.2217	0.1667	0.1217	0.8867



Jumps in asset prices

- Some commodity prices such as gas and electricity ***exhibit jumps.***
- **HEADS UP ON *STOCHASTIC CALCULUS:*** A process can be described by a ***stochastic differential equation***:

$$d\ln S = [\theta(t) - a\ln S]dt + \sigma dz + dp$$

where dp is a ***Poisson process generating jumps.***

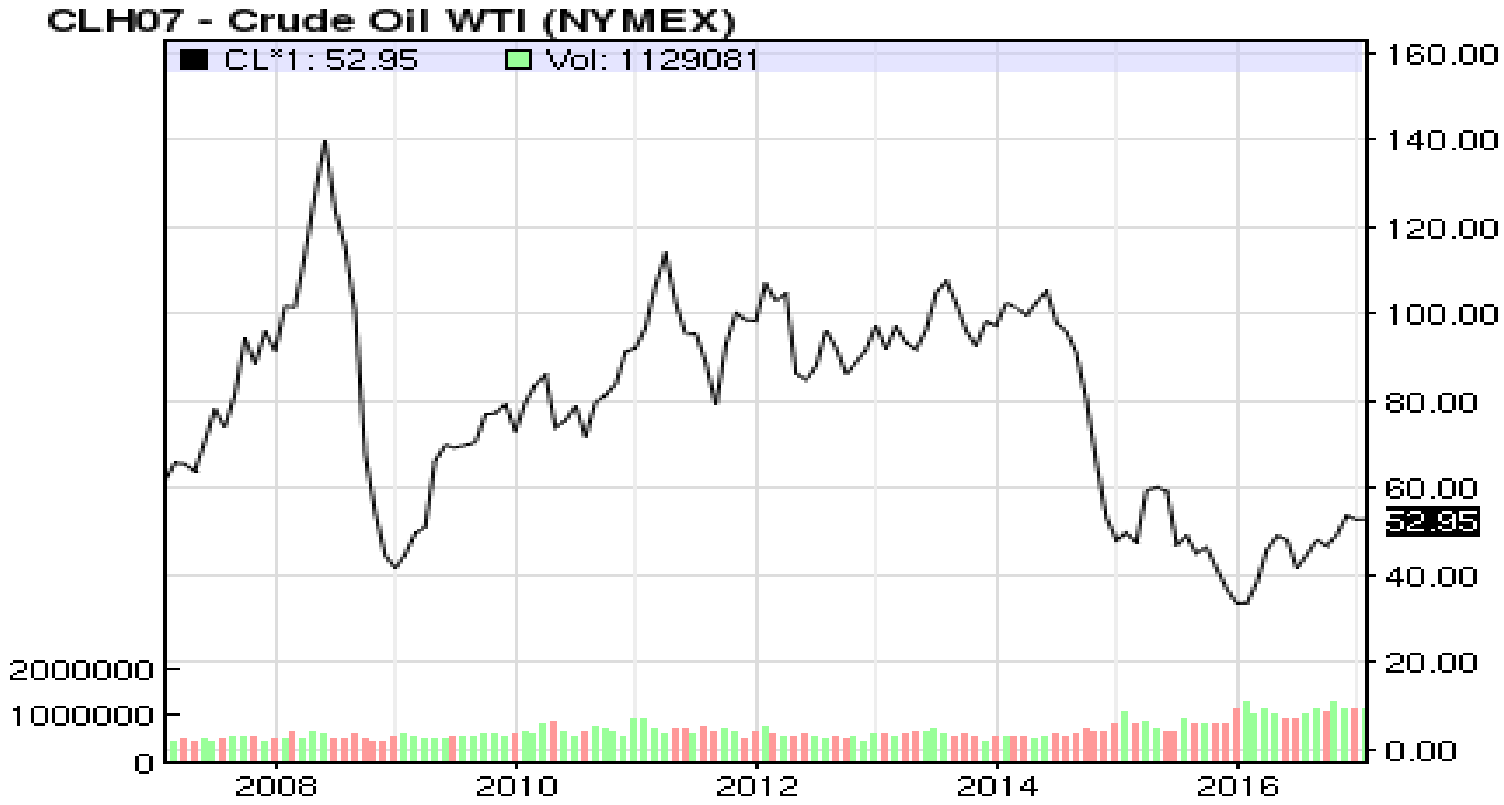
- If Poisson process is known we can use tree to model process without jumps and thereby determine $\theta(t)$.
- Can be implemented with ***Monte Carlo simulation.***

Stock price index dynamics

The S&P/TSX Composite Index
As of 07 March 2014

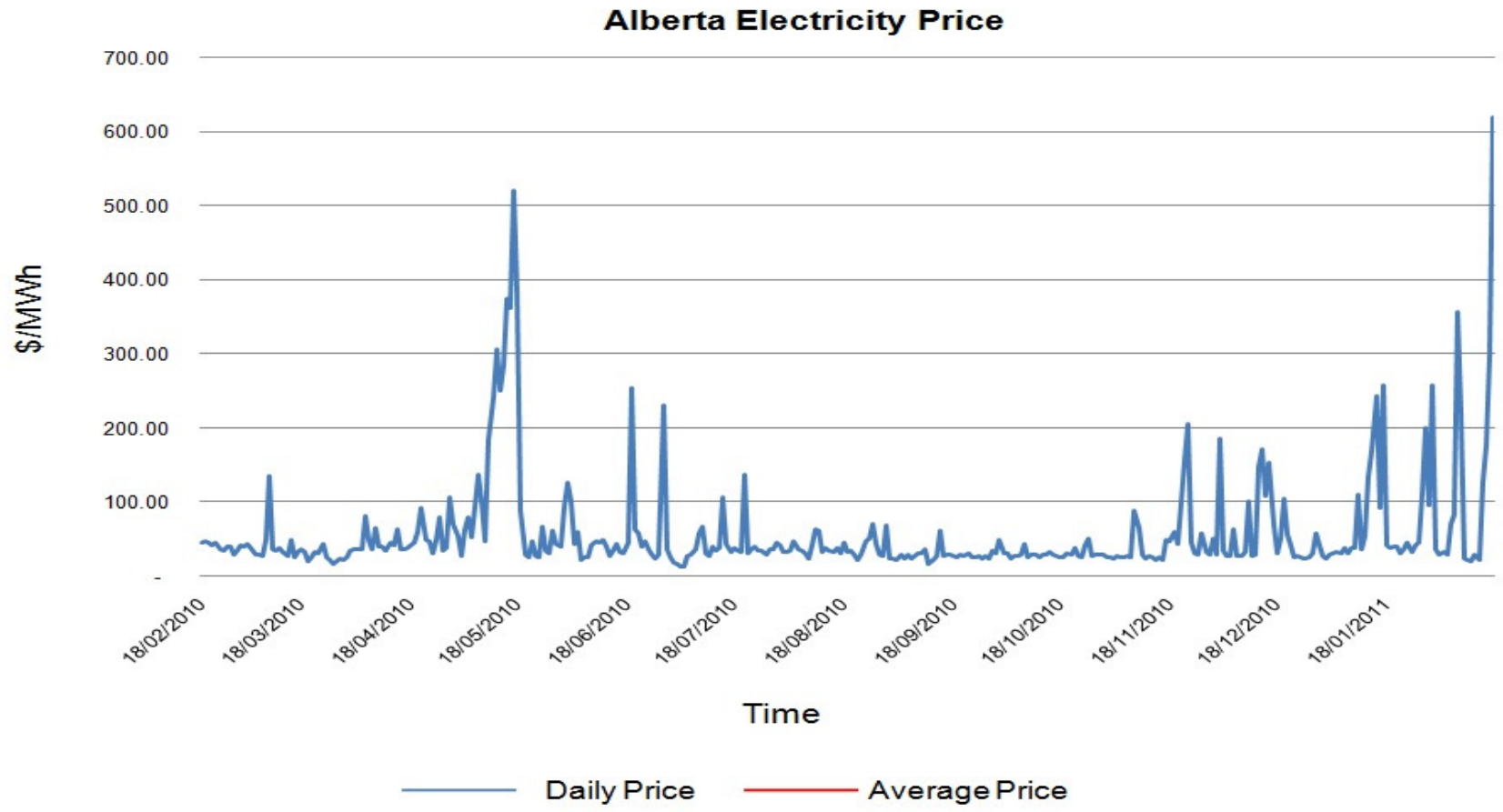
Mean-reverting property of prices is not essential!

Oil price dynamics



***Mean reversion* and *stochasticity* are the most *salient* features**

Electricity price dynamics



Dominant features: ***randomness***, ***mean-reversion***, ***seasonality*** and ***spikes***

Funding climate change adaptation: Weather Derivatives

- Heating degree days (HDD): For each day this is max(0, **65** – A) where A is the average of the highest and lowest temperature in ºF.
- Cooling Degree Days (CDD): For each day this is max(0, A – 65).
- Contracts specify the weather station to be used.

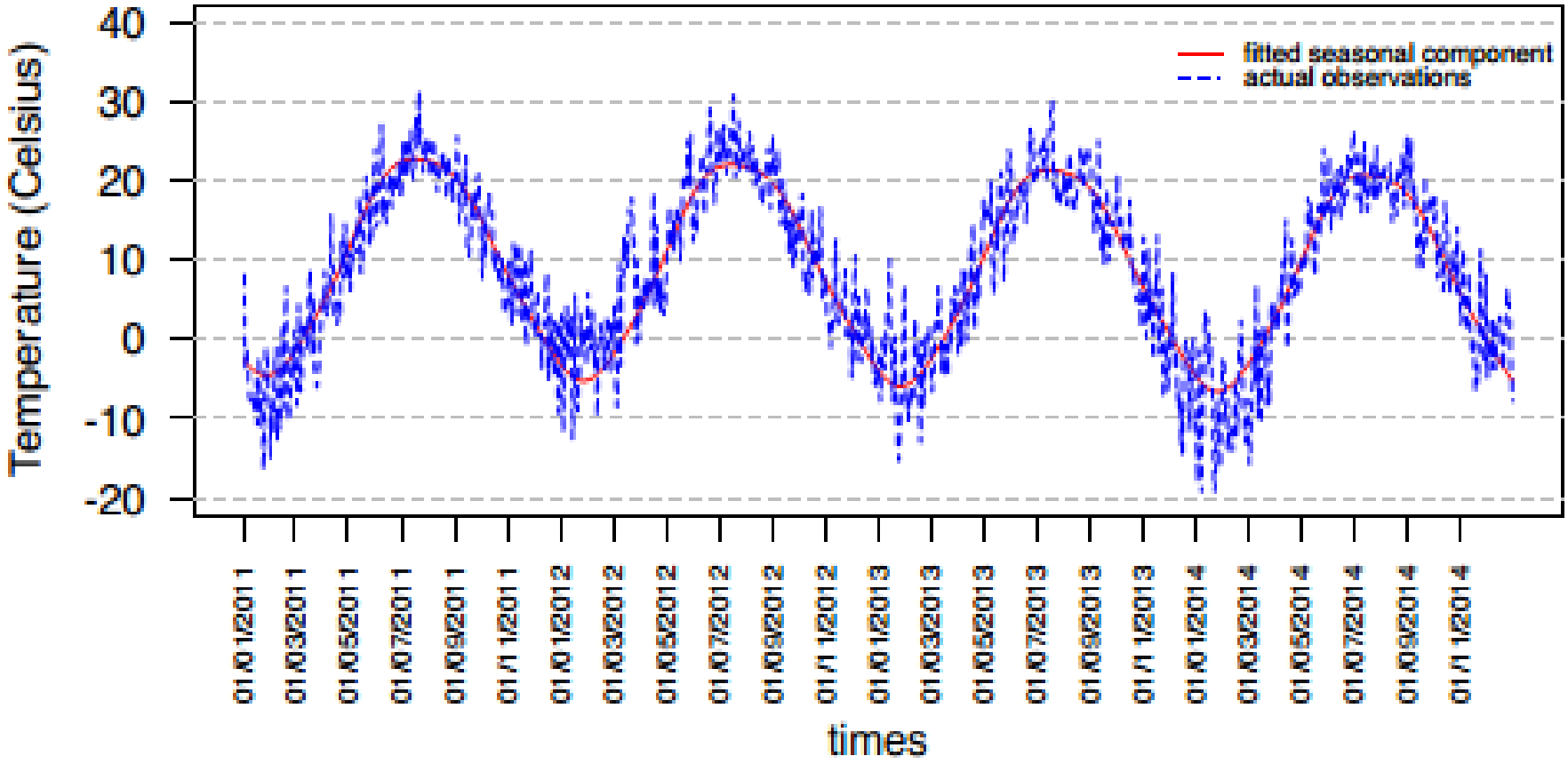
Day's HDD is a measure of volume of energy required for heating.

Day's CDD is a measure of volume of energy for cooling.

NOTE: Payment rate is $10,000 per degree on *cumulative CDD or HDD* over the term of the contract.

Example of a temp series to be modelled

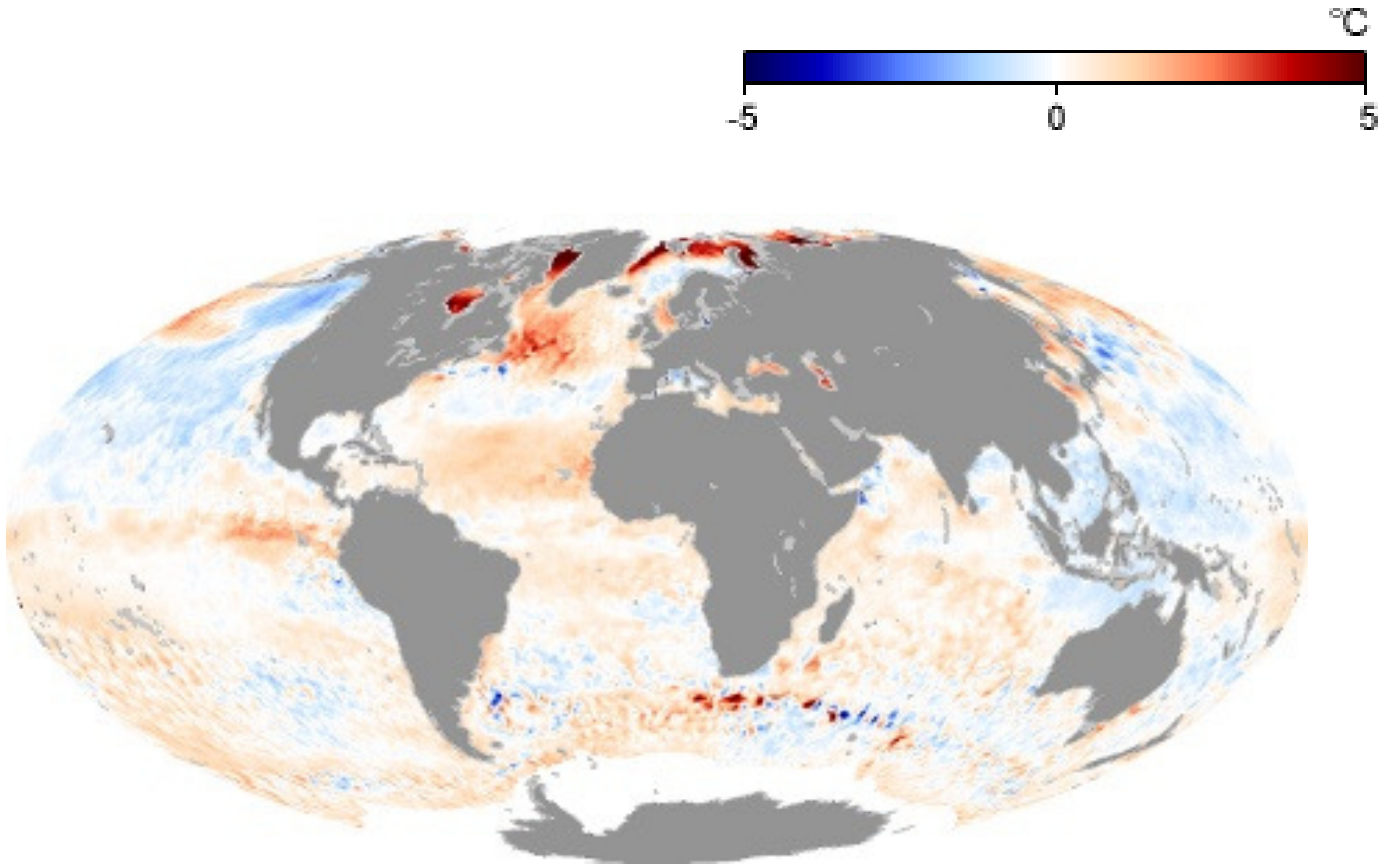
Temperature data at Toronto's Pearson Int'l airport collected by the Nat'l Climactic Data Centre (NCDC)



Sea surface temperature anomaly

Animation (from 2002 to 2011) at URL:

http://earthobservatory.nasa.gov/GlobalMaps/view.php?d1=AMSRE_SSTAn_M



Option on Sea-Surface Temperature: APPLICATION TO AQUACULTURE

- **Tasmania, Australia:** A European put option, based on forecasts of sea surface temperature (SST) that can be exercised at maturity, will pay out $100 for each degree *below* 18 °C. ***[Investors or option holders are growers/breeders of fish that survive in warm waters (***18 °C and above***).]***

- If summer temperatures rise *above* the 18°C threshold, a contracted aquaculture company (seller) **would not be obliged** to make a pay-out (they will keep the premium). The holders of the contract are fishers who do not have to deal with declining temperatures.

Weather/climate-driven mishap with devastating effect to AGRICULTURE

A mistimed rainstorm can ruin any hope of a profitable harvest.

Insurance Derivatives

- Crop insurance ***(contingent on the measurable impact of rain, drought, flood, pestilence, wind, and other natural calamities)***

WHAT ABOUT OTHER DISASTERS?

EARTHQUAKES, VOLCANIC ERUPTIONS, FLOODS, HAIL STORMS, DROUGHTS, *TSUNAMI*, CYCLONES, TYPHOONS, BLIZZARDS, TORNADOES, ETC,

- Catastrophe (CAT) bonds are an alternative to traditional reinsurance. If claims of a certain type are in a certain range, the interest and possibly the principal on the bond are used to meet claims (***based on impact of various disaster risks***).

Binomial model for asset price: From discrete to continuous time

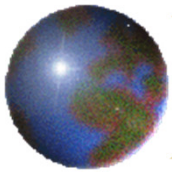
MODEL'S BUILDING BLOCK:

This is based on the concept of **random walk** (discrete time), which is a stochastic process described by

$$W_{(i+1)/n} = W_{i/n} + \frac{\varepsilon_i}{\sqrt{n}}, \qquad \varepsilon_i \sim N(0,1)$$

$$W_0 = 0.$$

As n → ∞, *W* becomes a **Brownian motion** (in continuous time).

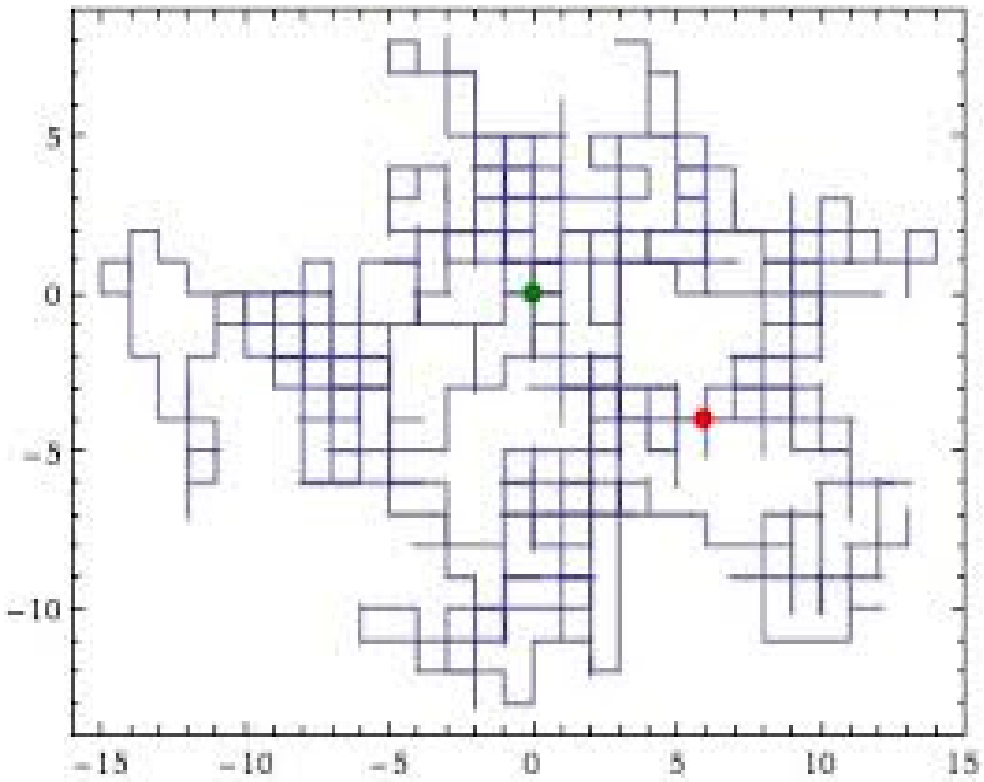


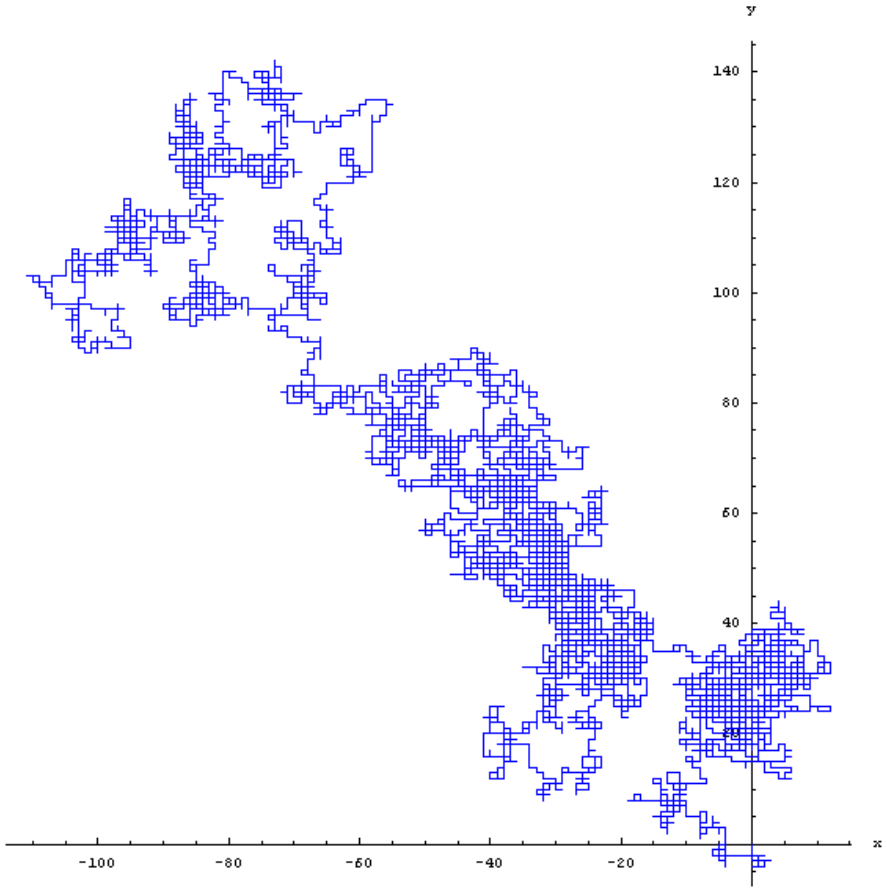
Sample paths of random walks can be simulated

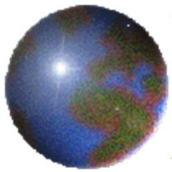
in various statistical/mathematical software or packages such as Scilab, Matlab, Mathematica,and even in EXCEL.

Show example in EXCEL.

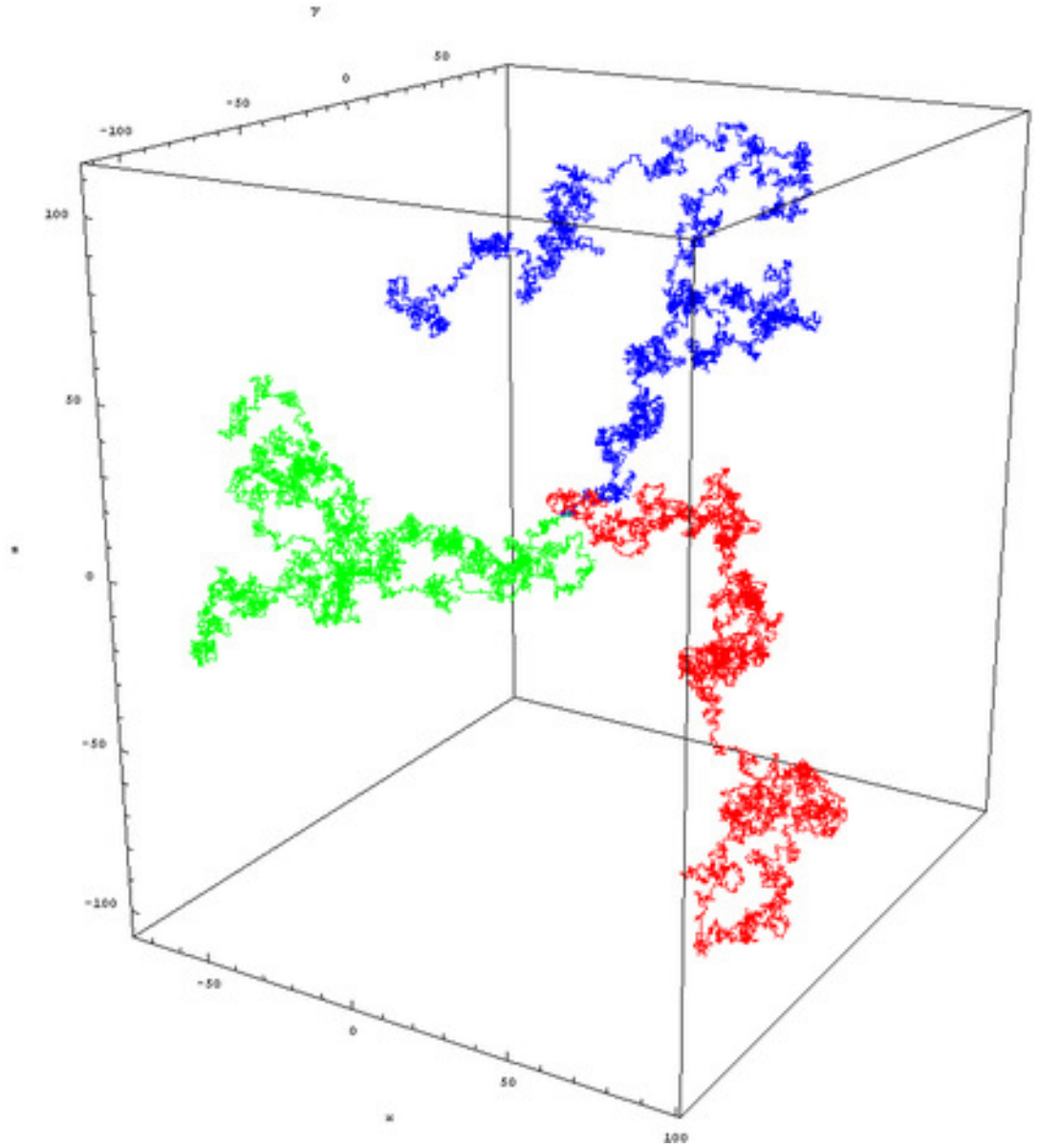
Random walk or drunkard's walk (in 2 dimensions)

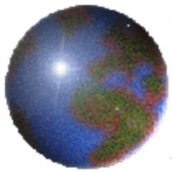






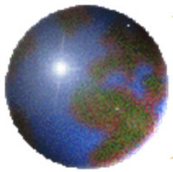
A random walk in 3 dimensions





BINOMIAL OPTION PRICING

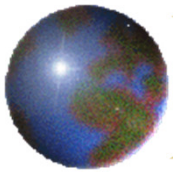
- Analytic solution in combinatorial form (DISCRETE TIME)
- CDF of a standard Gaussian/normal random variable (CONTINUOUS TIME) – ***BLACK-SCHOLES-MERTON OPTION PRICING FORMULA***



Options approach as a <u>way of thinking</u> in valuation of business ventures and technologies

Projects and investment opportunities include, but not limited to:

- Scientific research and development (valuation of patents, innovation and discovery)
- Infrastructure projects (bridges, roads, buildings, etc)
- New businesses and alternative ventures
- Drug/pharmaceutical discoveries
- Mining, oil-field explorations, etc
- Real-estate developments

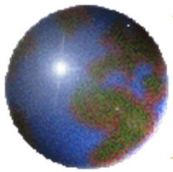


Application to Evaluation of Investment Opportunities AND Valuation of Technologies

- ***ABANDONMENT OPTION*** – option to sell or close down a project. *This is an American put option on the project's value.*
- Strike price = liquidation or re-sale value of projects less closing-down cost.
- Mitigate impact of poor investment outcome.

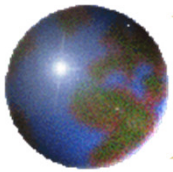
Options Approach: Assessing Investment Opportunities AND Valuation of Technologies (cont'd)

- ***EXPANSION OPTION*** – option to make further investments and increase outputs if conditions are favourable.
- *This is an American call option on the value of additional capacity.*
- Strike price = cost of creating additional capacity.



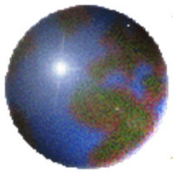
Application to Evaluation of Investment Opportunities AND Valuation of Technologies

- ***CONTRACTION OPTION*** – option to reduce the scale of a project's operation.
- *This is an American put on the value of the lost capacity.*
- Strike price = PV of future expenditures SAVED as seen at the time of the exercise of the option.



Application to Evaluation of Investment Opportunities AND Valuation of Technologies

- ***OPTION TO DEFER*** – option to wait or defer a project.
- *This is an American call option on the value of the project.*



Application to Evaluation of Investment Opportunities AND Valuation of Technologies

- ***OPTIONS TO EXTEND*** – option to extend the life of an asset by paying a fixed amount.
- *This is an European call option on the asset's future value.*

OPTIONS AS A WAY OF THINKING

- ***WHEN MAKING IMPORTANT DECISIONS IN LIFE!***