Maths 190 (Math'l Methods in Finance) – Week of 20 – 24 February 2017

SUMMARY OF IMPORTANT POINTS DISCUSSED IN THE LECTURE

The following concepts were covered/reviewed:
BASIC ELEMENTS OF STOCHASTIC PROCESSES RELEVANT TO DERIVATIVE PRICING

1. A **probability space** is a triplet $(\Omega, \mathcal{F}, P)$, where
   Ω is the set of all possible outcomes of a random experiment;
   $\mathcal{F}$ is a σ-**field** or $\sigma-$**algebra**, i.e., it is a non-empty collection of subsets of Ω and it is closed under set complementation and countable union; and
   P is a **probability measure**, i.e., $P : \mathcal{F} \to [0, 1]$ and P is countably additive.

2. A **random variable** (RV) X is a function where $X : \Omega \to I\!R$. More formally, X is a RV if $X^{-1}(B) \in \mathcal{F}$ for some Borel set B. A **Borel set** is any set that can be formed from open sets (or, equivalently, from closed sets) through the operations of countable union, countable intersection, and relative complement.

3. A **stochastic process** $X_t(\omega)$ or $X(t, \omega)$ is a family of RVs, where $\omega \in \Omega$ and t is a time index that could either be discrete or continuous.

4. The concept of probability measure for a discrete RV was also discussed. Intuitively, this refers to a set of probabilities.

5. We continued the discussion of **basic elements of stochastic processes relevant to derivative pricing**.

6. The filtration process. A **filtration** represents the history of a price process. More formally, a filtration is a sequence of non-decreasing sub-$\sigma-$fields or sub-$\sigma-$algebras (or collection of subsets of Ω). Some examples were given in the lecture to illustrate the concept of $\sigma-$field and filtration.

7. ***All models that are considered in finance are assumed to be defined on a probability space equipped with a filtration.***

8. A **contingent claim** on the binomial tree is a function of the nodes at a claim-time horizon T. Note that this is also a function of the filtration $\{\mathcal{F}_T\}$. Claims can either be (sample) path-dependent or path-independent.

9. **Conditional expectation** operator $E^P[\ \cdot\ |\mathcal{F}_k]$: This extends the idea of expectation to two parameters, namely a measure P and a history up to time k. This is an expectation along the latter portion of paths which have initial segment $\{\mathcal{F}_k\}$; if it makes it easier you may view the node attained at time k as the new root of the binomial tree and take expectations of future claims from there.