

# Maths 190 (Math'1 Methods in Finance) – Week of 23–27 January 2017

## SUMMARY OF IMPORTANT POINTS DISCUSSED IN THE LECTURE

The following concepts were covered/reviewed:

1. We discussed what contributed to the prolific increase of markets for derivative products, which are the need to control risk associated with foreign exchange and interest rate uncertainty in recent years.
2. The introduction of new products has become easier and less costly due to the emergence of new exchanges and market places as well as the deregulation of financial services in the 1980's.
3. The basics of derivatives and some examples were presented. A **derivative** is a contract between two parties that specifies condition – in particular, dates and the resulting values of underlying variables – under which payments or **payoffs** are to be made between the parties.

For example, a social security is a derivative which requires a series of payments from an individual to the government before age 65, and payoffs after 65 from the government to the individual as long as the individual remains alive. In this case, the pay-offs occur at predefined dates and depends on the individual's survival. Anyone who has ever taken out a mortgage with a prepayment privilege has perhaps unwittingly dabbled in derivatives. Insurance contracts (e.g., earthquake insurance) is a derivative in which individual make regular annual payments in exchange for a potentially much larger pay-off from the insurance company should an earthquake destroy his property.

4. Derivatives are also known as **contingent claims** since their pay-offs are “contingent” upon the outcome of an underlying variable. Derivatives have long existed with specific events or commodity prices as underlying variables. The big explosion of interest in derivatives, however, occurred only after purely financial derivatives with stock prices, stock indices, foreign exchange rates, bond prices and interest rates became the variables determining the size of the pay-off. Historians searching for a starting date might look to 1972, the formation of the International Monetary Market (IMM), a division of the Chicago Mercantile Exchange (CME), or April 1973, the opening of the Chicago Board Options Exchange (CBOE), the first modern exchanges to trade financial derivatives.
  
5. We also studied assets underlying exchange-traded derivatives. They usually fall into the following categories: commodities, stocks/equities, stock market indices, currencies, fixed-income securities or interest rate-dependent products.
  
6. The first exchange-traded derivatives, which traded on the Chicago Board of Trade, (CBOT) had commodities as underlying assets. With the creation of CBOT in 1948, agricultural commodities, particularly corn and wheat, became the first underlying assets to have exchange-traded futures in the US. Until the last three decades, these were the most actively traded derivatives. Interest in them arises principally from farmers needing to hedge both their costs and their revenues. In addition, food processors, storage firms, domestic exporters and foreign importers also use these derivatives to hedge their exposure to prices. For example, exchange-traded options and futures permit hedging of both crude and refined oil (heating oil or gasoline). This permits refiners to hedge both their costs (by buying futures) and revenues (selling futures).

Certain underlying assets for derivatives are also classified as belonging to either cash-and-carry markets or price-discovery markets. A discus-

sion of these markets and some examples were given in the lecture.

7. The most popular exchange-traded index derivatives have the S&P 500 index as the underlying asset. The S&P 500 index consists of 500 large capitalisation stocks, comprising about 80-85% of the market value of all stocks traded on the New York Stock Exchange (NYSE). The index is constructed first by calculating the concurrent market value of each of the 500 stocks (current market price per share times number of shares outstanding). These values are then added together to obtain the total market value of all outstanding shares in the index.
  
8. In the modern world, the archetypical example of “cash” is a short-term US **Treasury bill** (T-bill). These securities, issued and guaranteed by the US government, are zero-coupon bonds since they pay no coupons and only provide payment of principal at maturity. T-bills are also issued by the government of Canada. A **repurchase agreement** (repo), another candidate for cash, is a sale of a government fixed income securities to a ‘lender’ with an agreement to buy them back in the future. With the lender holding the borrower’s securities as collateral, losses due to default is minimal. If the securities are T-bills, then the borrower must agree to repurchase them at higher price. **Eurodollars** are deposits of US dollars in a bank outside the United States. Coupon bonds issued by the US Treasury (or Canadian Treasury) are called **T-notes** (with an initial maturity of 10 years or less) and **T-bonds** (with an initial maturity of more than 10 years).
  
9. The largest cash markets (to the US) currencies are the Japanese yen, Euros, British pound, Swiss francs and Canadian dollars. Trading is largely over-the-counter with banks serving as intermediaries. Transfers take the form of book entry, so they do not require the physical transfer of the currency.

10. Derivatives are defined by the timing of and other conditions for their pay-offs. There are four logically pure timing patterns for the payment for and receipt of an asset as illustrated in the securities classification matrix shown in class. In an ordinary *cash transaction*, the asset is both paid for and received in the present. In contrast, *borrowing* money effectively allows the borrower to purchase an asset (with the borrowed funds) in the present, but pay for it in the future (by repaying the loan). *Lending* money permits just the opposite. Note that the amounts and timing of the payments in these cases are completely determined in advance. Finally, in a *forward transaction* (and nominally futures transaction), both payment and receipt are delayed until the same future date, but (and this is critical) the price to be paid and the time of payment are preset in the present.

Forward contracts are pervasive and several examples were given in the lecture. Forward transaction may be preferred by someone who aims to eliminate the uncertainty in future prices of an underlying asset. A trader could arrange a forward transaction to lock in a preset price. Such trader is called a **hedger**. Before taking a forward position, a hedger already holds its underlying asset, or has a pre-commitment to receive or deliver the underlying asset. A **speculator** uses forward transactions to take on risk. He participates in a forward transaction without any existing position or pre-commitment in the underlying asset. An **arbitrageur** takes offsetting positions in two or more financial instruments (a forward contract could be one of them) to lock in a profit.

11. A standard **forward contract** is an agreement to buy or to sell an underlying asset at a predetermined price during a specified future period, where the terms are initially set such that the contract is costless. At the inception of a forward contract, no money changes hands; the actual trade is postponed until a pre-specified future period when its underlying asset is exchanged for cash.

For example, for a corn forward contract, an agreement may be made today to exchange \$10,000 in six months (**time-to-delivery**) for 5,000

bushels of corn of a prespecified grade delivered at a pre-specified warehouse. The prearranged price of \$10,000 is called the **delivery price**. This price is not to be confused with the current value at the inception of the forward contract itself. There are two counterparties: a **buyer** and a **seller**. The buyer is obligated to pay \$10,000 in six months from now to the seller; in return, the seller is obligated to deliver the 5,000 bushels of corn of the agreed grade six months from now to the buyer at the agreed location.

12. A **futures contract** is similar to a forward contract except that is resettled at the close of trading each day. At that time, a new futures price is set that resets the present value of the futures contract to zero; and any difference between the successive futures prices is made as a cash payment between the parties. Therefore, if the futures price rises, the difference is received by the buyer and paid to him by the seller; if the futures prices falls, the difference is received by the seller and paid to him by the buyer.
  
13. A standard forward contract is an agreement to make a single future exchange of a fixed payment for an asset of uncertain future value. A standard **swap** is an exchange of a sequence of cash flows from two assets without necessarily exchanging the assets themselves. That is, a swap can often be decomposed into a portfolio of forward contracts with a sequential series of delivery dates.

The swap market developed because two investors might find that, whilst they may have had a comparative advantage in borrowing in one market, they were at a disadvantage in another market in which they wanted to borrow. If these markets were counter-matched by the two parties, the two could get the best of both worlds through a swap.

14. Options are a more complex class of derivatives. A standard **option** is an agreement either to buy or to sell an underlying at a predetermined

price on or before a specified date in the future, where one and only counterparty can cancel the agreement.

A standard option is similar to a forward since it is also a contract for an exchange in the future, where the price to be is preset. For options, this preset price is called the **strike price** or **exercise price**, the time to the last date the exchange can take place is termed the **time-to-expiration** and this date is called the **maturity**. However, an option differs from a forward since one of the counterparties can cancel it. The party is termed the **buyer** of the option; the other party who must honour the agreement if the buyer wishes to exercise is termed the **seller** (or “writer”). Since for the buyer, an option is a right, not an obligation, he will choose to cancel the option if it turns out not to be to his benefit. On the other hand, the option seller has no such right and must honour the contract if the buyer chooses to **exercise** his option. Such a right is generally of some value. So the option buyer must pay something to the seller for this right in the present (the **option price** or **premium**), although usually the bulk of the cash transaction, if it occurs, happens in the future.

There are two basic types of standard options, depending on which counterparty, the one who is to receive or the who is to deliver the underlying asset, has the right to cancel the contract.

If the party who can buy the underlying asset has the right to cancel, the option is termed a **call**; if the party who can sell the underlying asset has the right to cancel, the option is termed a **put**.

15. When you trade derivatives, an important aspect to keep in mind is that there two counterparties. Ignoring broker-dealers and governments, each counterparty can gain dollars only at the other’s expense. We call this a **zero-sum game** since no matter what happens the summed dollar gains and losses across the parties is zero. When you think that buying a forward, call or put is a good idea, remember that someone else must believe that selling the same derivative is also a good idea. Therefore, it may be equally important for you to understand why you

want to buy the derivative as it is for you to understand why your counterparty wants to sell.

16. A number of reasons why one would use derivatives instead of their underlying were discussed in class. These include the following:

- Derivatives are occasionally available on underlying variables which are not themselves traded; in that case, the derivative may offer a way to gain financial exposure to or protection against the underlying variable.
  
- Derivatives can create customised pay-off which are better suited to an investor's preference.
  
- Derivatives often permit investors to take more efficient advantage of certain types of information, such as information about pay-outs (for example, cash dividends in the case of options on stocks).
  
- With a smaller amount of money than the price of the underlying asset, derivatives can provide a similar exposure to movements of the underlying asset.
  
- Derivatives can provide a similar exposure to the underlying asset price, but at reduced trading costs, although this usually applies only for short-holding periods.
  
- To the extent that derivatives represent implicit borrowing or lending, these implicit positions can often be established at more favourable interest rates than would be ordinarily available to an investor.
  
- Derivatives may offer special tax advantages, or permit investors to achieve pay-offs that regulations would not permit solely by using their underlying assets.

17. Compared to forwards, futures and swaps, options are appropriate in

the following circumstances:

- The investor prefers non-symmetric pay-offs; that is, where the downside pay-offs and upside pay-offs have different sensitivities to movements in the underlying asset price.
- The investor has special information concerning the shape of the subjective probability distribution of the future underlying asset price, even though he has no special information concerning its location (expected future price); or the investor desires to hedge against changes in the distribution's shape.

18. Options on stocks, currencies, stock indices and futures are traded on exchanges. Many of these exchanges together with their features and mechanics were presented in the lecture. Refer to the slide presentation discussed on 26 Jan 2017.

19. The value at maturity or *terminal value* of a long position in a forward contract on one unit of an asset is  $S_T - K$  where  $K$  is the delivery price and  $S_T$  is the spot price of the asset at maturity of the contract.

Similarly, the terminal value of a short position in a forward contract on one unit of asset is  $K - S_T$ . Since *it costs nothing to enter into a forward contract*, the terminal value of the contract is also the trader's total profit/loss from the contract. The profit diagrams from certain types of forward contracts were illustrated in the lecture.

Under the forward contract, the whole gain or loss is realised at the end of the life of the contract. Under the futures contract, the gain or loss is realised day by day because of the daily settlement procedures.

20. Options are fundamentally different from forward and futures contracts. An option gives the holder the right to do something. The holder does not have to exercise this right. By contrast, in a forward or futures



contract, the two parties have committed themselves to do something. Whereas it costs nothing to enter into a forward contract, the purchase of an option requires an up-front payment.

A call option gives the holder the right to buy an asset by a certain date for a certain price. A put option gives the holder the right to sell an asset by a certain date for a certain price. Options are either American or European. This has nothing to do with geographical location. *American options* are options that can be exercised at any time up to the expiration date, whereas *European options* are options that can only be exercised on the expiration date itself. Most of the options that are traded on exchanges are American. However, European options are generally easier to analyse than American options, and some properties of an American option are frequently deduced from those of its European counterpart.