

# Maths 190

Topics be covered for the week of 20-27 February 2017

Long Exam 1: Wednesday, 22 Feb 2017 from 09:00-11:00HRS

For this week, we aim to cover the following theories/concepts:

1. Futures options or options on futures (**to be included in Long Exam 1**). *This was mentioned in the previous discussions but was not fully treated. Hence, this is revisited.*
2. Elements of discrete-time stochastic process. These would include the discussion of:
  - (a) probability space
  - (b) random or stochastic process
  - (c) probability measure
  - (d) filtration
  - (e) general contingent claims
  - (f) conditional operator/conditional expectation
  - (g) previsible process
  - (h) martingales
  - (i) law of iterated expectations (or tower law)
3. Random walk model
4. Brownian motion  $W_t$

## Calculus of Brownian motion

5. Analysing the qualitative behaviour of  $W_t$ , its properties and how it's being employed in financial modelling

6. Stochastic differential equation of the form  $dX_t = \mu(X_t, t)dt + \sigma(X_t, t)dW_t$
7. Itô's lemma/Itô's differentiation rule
8. The “multiplication rule”
9. The geometric Brownian motion as a model for price dynamics
10. Some applications and examples
11. Simulation of random walk and its convergence to Brownian motion
12. Basics of Monte-Carlo simulation. Application to valuation of options and implementation in the Excel software will be considered.
13. Partial differential equation (PDE) approach in valuation: The intent here is to derive the PDE, with terminal condition, that satisfies the price of a derivative security. Here, we shall consider a portfolio whose value is  $\Pi$  in which the holder is short one derivative security and long an amount of  $\frac{\partial f}{\partial S}$  of shares, where  $S$  is the price of the underlying asset and  $f$  is the price of a derivative security.

14. The Feynman-Kac's result linking the solution of a PDE to conditional expectation will be presented without proof