Maths 190
Topics be covered for the week of 27 February - 03 March 2017

For this week, we aim to cover the following theories/concepts:

1. Elements of discrete-time stochastic process **(Continuation)**
   These would include the discussion of:
   (a) probability space
   (b) random or stochastic process
   (c) probability measure
   (d) filtration
   (e) general contingent claims
   (f) conditional operator/conditional expectation
   (g) previsible process
   (h) martingales
   (i) law of iterated expectations (or tower law)

2. Random walk model

3. Brownian motion W_t

   Calculus of Brownian motion

4. Analysing the qualitative behaviour of W_t, its properties and how it's being employed in financial modelling

5. Stochastic differential equation of the form $dX_t = \mu(X_t, t)dt + \sigma(X_t, t)dW_t$

6. Itô's lemma/Itô's differentiation rule

7. The "multiplication rule"

8. The geometric Brownian motion as a model for price dynamics

9. Some applications and examples

10. Simulation of random walk and its convergence to Brownian motion

11. Basics of Monte-Carlo simulation. Application to valuation of options and implementation in the Excel software will be considered.

12. Partial differential equation (PDE) approach in valuation: The intent here is to derive the PDE, with terminal condition, that satisfies the price of a derivative security. Here, we shall consider a portfolio whose value is Π in which the holder is short one derivative security and long an amount of $\frac{\partial f}{\partial S}$ of shares, where S is the price of the underlying asset and f is the price of a derivative security.

13. The Feynman-Kac's result linking the solution of a PDE to conditional expectation will be presented without proof