Financial Modelling 9561B
Winter 2014

Assignment No.2

GUIDELINES ON SUBMITTING ASSIGNMENTS

- Your assignment paper must include the Marking Scheme as a cover page. This marking scheme can be downloaded from the course website. **Failure to follow this instruction can result to a 2-point deduction on your assignment mark.**
- YOU MUST WRITE YOUR OWN WORK IN YOUR OWN WORDS, using full sentences and proper English grammar. It is your responsibility to familiarise yourself with the provisions of the University Regulation concerning academic integrity and honesty. **Any behaviour that can potentially lead to plagiarism, cheating and copying from/sharing with another student answers in an assignment or exams is a serious offence and carries with it severe penalty.** Do not take this warning lightly; academic penalties have dire consequences on your future studies and career.
- Do not submit your rough work! Do the problem set and then re-write it at least once - neatly, with adequate amount of clear explanation. The rewriting stage is the most important one for finding errors in one's work, and it will also deepen your understanding of the subject matter. Assignments are marked for both technical correctness and elegance of presentation.
- Bear in mind to include a sufficient amount of explanation about your work so that any marker does not have to guess what you mean. The grader of your work will determine if you understand what you are writing, not merely that you reach the particular correct answer.
- On questions where a computer output is required or deemed necessary, include the output in the text of your answer at the appropriate locations - do not put it all in a bunch at the end of your assignment. Unless, you are instructed to submit your work in a CD or disc, you are expected to hand in a PRINTED COPY.

Do as indicated. ENJOY!

1. [4 points]
   Solve the following boundary-value problem in the domain $[0,T] \times I\!R$.

$$\begin{aligned} \frac{\partial F}{\partial t} + \mu x \frac{\partial F}{\partial x} + \frac{1}{2}\sigma^2 x^2 \frac{\partial^2 F}{\partial x^2} &= 0, \\ F(T,x) &= \ln x^2. \end{aligned}$$

   Here, μ and σ are assumed to be known constants. (Hint: Use a stochastic representation result.)

2. [4 points]
   Given a probability space $(\Omega, \mathcal{F}, P)$, suppose S_t follows the dynamics of a geometric Brownian motion with drift μ and volatility σ. Consider the wealth process Z_t of an agent, which has the differential form $dZ_t = \Delta(t)dS_t + r_t(Z_t - \Delta(t)S_t)dt$ where Δ is the number of risky assets to buy and hold at time t and r is the risk-free rate. Let $\beta(t) = \exp\left(\int_0^t r_u du\right)$ be the compounding factor process. Show that the dynamics of $\frac{Z_t}{\beta(t)}$ is given by $d\left(\frac{Z_t}{\beta(t)}\right) = \Delta(t) d\left(\frac{S_t}{\beta(t)}\right)$ and argue that the discounted wealth process is a martingale under a measure Q, where Q is linked to the measure P via the Girsanov density described in the lecture.

3. [8 points]
   (a) Consider the Vasiček model $dr_t = (\alpha - \beta r_t)dt + \sigma dW_t$ for the interest rate process r, where α, β and σ are positive constants. Write down the zero-coupon bond price for the Vasiček model in terms of the parameters α, β and σ. Give the bond price under the Ho-Lee model (i.e., the case when $\beta = 0$) by considering it as a special case of the Vasiček model when $\beta = 0$. [4 pts]

(b) Consider again the Ho-Lee dynamics for the interest rate process r_t. Find the distribution of r_t and use this distribution to find the bond price for the Ho-Lee model. Ensure that both bond prices in (a) and (b) reconcile. [4 pts]

4. [4 points]
Let $X_t = [X_t^1, X_t^2]^\top$ be a two-dimensional process and each X_t^i, $i = 1, 2$, is driven by a three-dimensional Brownian motion $W_t = [W_t^1, W_t^2, W_t^3]^\top$. Suppose $X_t^1 = W_t^1 + W_t^2 + W_t^3$ and $X_t^2 = (W_t^2)^2 - W_t^1 W_t^3$. Show that X_t satisfies the dynamics

$$dX_t = \begin{bmatrix} 0 \\ 1 \end{bmatrix} dt + \begin{bmatrix} 1 & 1 & 1 \\ -W_t^3 & 2W_t^2 & -W_t^1 \end{bmatrix} \begin{bmatrix} dW_t^1 \\ dW_t^2 \\ dW_t^3 \end{bmatrix}.$$

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