Statistical Sciences 9561B
Assignment No.3

Due Date: 09:30am, Wednesday, 02 April 2014
Put your assignment in the drop box, labelled FM 9561B, located across WSC 240. The instructor will pick up the assignment papers at exactly 9:30am. Under no circumstances will late assignments be accepted!

GUIDELINES ON SUBMITTING ASSIGNMENTS

- YOU MUST WRITE YOUR OWN WORK IN YOUR OWN WORDS, using full sentences and proper English grammar. It is your responsibility to familiarise yourself with the provisions of the University Regulation concerning academic integrity and honesty. **Any behaviour that can potentially lead to plagiarism including cheating and copying from or sharing with another student answers in an assignment or exam is a serious offence, and carries with it severe academic penalty.** Do not take this warning lightly; academic penalties have dire consequences on your future studies and career.

- Do not submit your rough work! Do the problem set and then re-write it at least once - neatly, with adequate amount of clear explanation. The rewriting stage is the most important one for finding errors in one's work, and it will also deepen your understanding of the subject matter. Assignments are marked for both technical correctness and elegance of presentation.

- Bear in mind to include a sufficient amount of explanation about your work so that any marker does not have to guess what you mean. The grader of your work will determine if you understand what you are writing, not merely that you reach the particular correct answer.

- On questions where a computer output is required or deemed necessary, include the output in the text of your answer at the appropriate locations - do not put it all in a bunch at the end of your assignment. Unless, you are instructed to submit your work in a CD or disc, you are expected to hand in a PRINTED COPY.

Do as indicated. ENJOY!

1. **[5 points]**
   Assume that the short-rate process r_t is given by the SDE

   $$dr_t = \mu(t, r_t)dt + \sigma(t, r_t)dW_t,$$

   where W_t is a Brownian motion under a risk-neutral measure. Furthermore, suppose that the bond price $B(t, T)$ under this modelling framework has an exponential-affine form, i.e.,

   $$B(t, T) = \exp(A(t, T)r_t - C(t, T))$$

   for deterministic functions $A(t, T)$ and $D(t, T)$.

   Show that a necessary condition for the bond pricing formula to have an exponential-affine form is for the risk-neutral drift and volatility of r_t to have the forms $\mu(r_t, t) = \alpha(t) + \beta(t)r_t$ and $\sigma_t = \sqrt{\gamma(t)r_t + \delta(t)}$. That is, the drift must be affine and volatility must be square-root affine. Here, the functions $\alpha(t),\ \beta(t),\ \gamma(t)$ and $\delta(t)$ are deterministic.

2. **[4 points]**
   Consider the Euler discretised version for the short-rate process r_t under the Ho-Lee model given by $dr_t = \theta dt + \sigma dW_t$. Find the maximum likelihood estimates of θ and σ.

3. **[4 points]**
   Suppose the volatility $\sigma(t, T)$ for the forward rate under the HJM set-up is given by $\sigma(t, T) = \vartheta e^{-\zeta(T-t)}$ for positive constants ϑ and ζ.
   (a) In not more than 15 words, explain the motivation for the proposed choice of $\sigma(t, T)$. [1 pt]

   (b) Show that this HJM's specification of $\sigma(t, T)$ is equivalent to the Hull-White model for the short-rate process. [3 pts]

4. [**4 points**]
A consol bond is a bond that forever pays a constant continuous coupon c. This means that over every interval with length dt the consol pays $c(dt)$. No face value is ever paid. Suppose further that the bond price dynamics under a martingale measure are given by

$$dB(t,T) = B(t,T)r_t dt + B(t,T)\sigma(t,T)dW_t,$$

where W_t is a standard Brownian motion under the martingale measure. If $G(t)$ denotes the price of the consol, find an expression for the volatility component of $G(t)$ (in terms of the bond price and its parameters).

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