FM 9561B - Fixed-Income Modelling
Outline of Lectures: 13 – 17 January 2014

For this week, we aim to cover the following theories/concepts:

Construction and properties of Itô's integral

1. The martingale property: We shall show "If $I(t)$ is an Itô integral then $I(t)$ is a martingale".

2. Itô isometry: Given the Itô integral $\int_0^t g(s)dW_s$, $E[I(t)^2] = E\left[\int_0^t g^2(s)ds\right]$.

3. Itô integral of a general integrand: The idea here is to consider a sequence of elementary processes $\{g_n\}_{n=1}^{\infty}$ such that $E\left[\int_0^T |g_n(t) - g(t)|^2 dt\right] \to 0$ as $n \to \infty$. We shall then define the Itô integral $\int_0^t g(s)dW_s$ as $\lim_{n\to\infty} \int_0^t g_n(s)dW_s$. The major challenge to this approach is to ensure that this limit exists.

4. We shall analyse if the Itô integral $\int_0^t W_s dW_s$ is well-defined.

5. Quadratic variation of an Itô integral.

6. Itô's differentiation rule. Derivation of Itô's formula.

7. Quadratic variation of a geometric Brownian motion.

8. Examples of Itô processes.

9. The generalised one-dimensional case of Itô lemma.

Some elements of financial valuation

10. Investor's wealth process and the pricing of a contingent claim

11. Description of an Itô process

12. Re-visiting the generalised Itô's lemma for one dimensional case

13. Delta-hedging, replicating portfolio and the PDE approach in option pricing

14. Feynman-Kac's theorem

15. Solving for the mean and variance of a stochastic process without having to solve the SDE. The method will be illustrated using the Cox-Ingersoll-Ross interest rate model

16. Cross-variations of BMs and the multi-dimensional Itô's formula

17. Change of measure (Girsanov's theorem) and applications in finance