MARKING SCHEME

Statistical Sciences 3520B – Winter 2014
Solutions to Assignment No. 1
(Questions assigned for submission)

♣Required Assignment Question 1 (Problem 1.32) [4 points]

On July 15, 2010, an investor owns 100 Google shares. Suppose the share price is $497 and a December put option with a strike price $460 costs $27.30. The investor is comparing two alternatives to limit downside risk. The first involves buying one December put option contract with a strike price of $460. The second involves instructing a broker to sell the 100 shares as soon as Google's price reaches $460 (this is called stop-loss order). Discuss the advantages and disadvantages of the two strategies. In particular, discuss under what circumstances will the option lead to a better outcome, and under what circumstances will the stop-loss strategy lead to a better outcome? 0.5

Answer: The second alternative, known as a stop or stop-loss order, costs nothing and ensures that $46,000, or close to $46,000, is realised for the holding in the event the stock price ever falls to $460. 0.5

The put option costs $2,730 and guarantees that the holding can be sold for $4,600 any time up to December. If the stock price falls marginally below $460 and then rises the option will not be exercised, but the stop-loss order will lead to the holding being liquidated. 1

There are some circumstances where the put option alternative leads to a better outcome and some circumstances where the stop-loss order leads to a better outcome. If the stock price ends up below $460, the stop-loss order alternative leads to a better outcome because the cost of the option is avoided. 1 If the stock price falls to $380 in November and then rises to $490 by December, the put option alternative leads to a better outcome. 1 The investor is paying $2,730 for the chance to benefit from this second type of outcome.

♣Required Assignment Question 2 (Problem 5.28) [4 points]

A bank offers a corporate client a choice between borrowing cash at 11% per annum and borrowing gold at 2% per annum. (If gold is borrowed, interest must be repaid in gold. Thus, 100 ounces borrowed today would require 102 ounces to be repaid in one year.) The risk-free interest rate is 9.25% per annum, and storage costs are 0.5% per annum. Discuss whether the rate of interest on the gold loan is too high or too low in relation to the rate of interest on the cash loan. The interest rates on the two loans are expressed with annual compounding. The risk-free interest rate and storage costs are expressed with continuous compounding.

Answer: Suppose that the price of gold is \$1000 per ounce and the corporate client wants to borrow \$1,000,000. The client has a choice between borrowing \$1,000,000 in the usual way and borrowing 1,000 ounces of gold. If it borrows \$1,000,000 in the usual way, an amount equal to $1,000,000 \times 1.11 = \$1,110,000$ must be repaid. If it borrows 1,000 ounces of gold it must repay 1,020 ounces. In equation (5.12) or formula derived in class, $r = 0.0925$ and $u = 0.005$ so that the forward price is

$$1000e^{(0.0925+0.005)\times 1} = 1102.41$$

By buying 1,020 ounces of gold in the forward market the corporate client can ensure that the repayment of the gold loan costs

$$1,020 \times 1102.41 = \$1,124,460$$

Clearly the cash loan is the better deal ($1,124,460 > 1,110,000$).

This argument shows that the rate of interest on the gold loan is too high. What is the correct rate of interest? Suppose that R is the rate of interest on the gold loan. The client must repay $1,000(1+R)$ ounces of gold. When forward contracts are used the cost of this is

$$1,000(1+R) \times 1102.41$$

This equals the \$1,110,000 required on the cash loan when $R = 0.688\%$. The rate of interest on the gold loan is too high by about 1.31%. However, this might be simply a reflection of the higher administrative costs incurred with a gold loan. It is interesting to note that this is not an artificial question. Many banks are prepared to make gold loans.

♣Required Assignment Question 3 (Not in Hull, 8th ed) [8 points]

In each case below provide a table (and plot) showing the relationship between profit/ loss and final stock price. Ignore the impact of discounting.

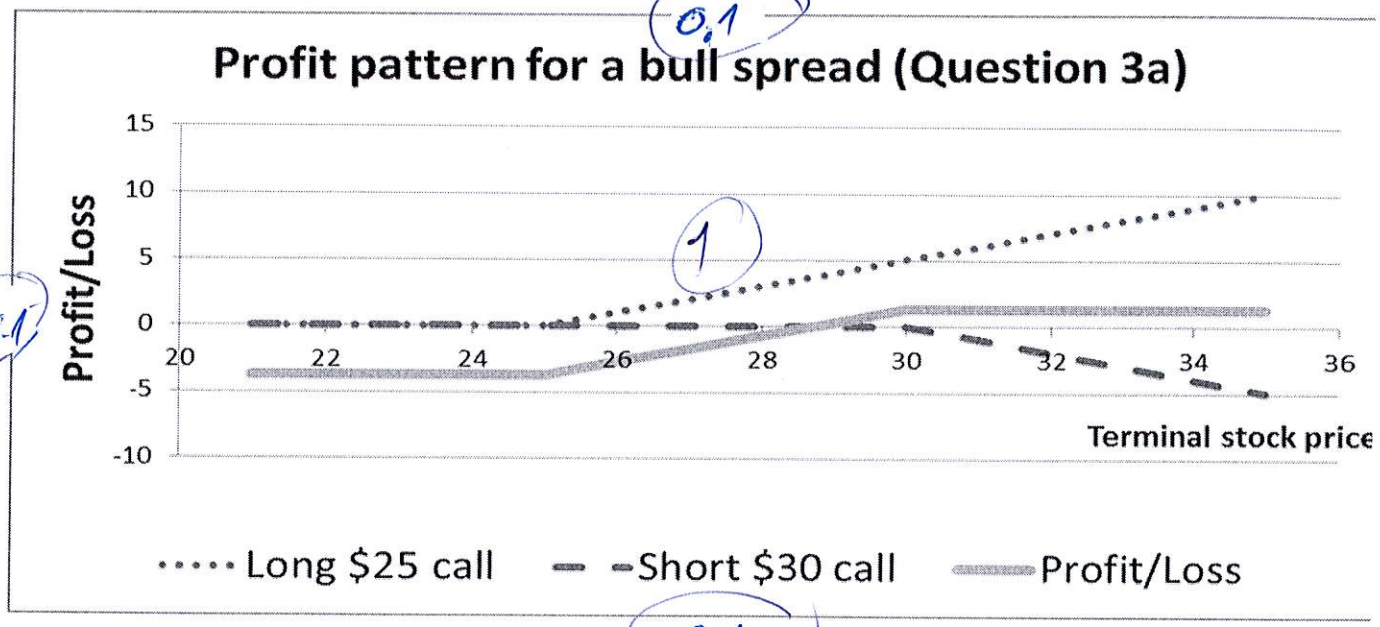
a. *Call options with strike prices of \$25 and \$30 cost \$7.90 and \$4.18, respectively. Both have maturity of six months. Demonstrate the profit/ loss pattern for a trading strategy (called bull spread) of buying the \$25 call and selling the \$30 call. Why would an investor be engaging such a strategy? [2.5 pts]*

Answer: A call option with a strike price of 25 costs \$7.90 and a call option with a strike price of 30 costs \$4.18. The cost of the bull spread is therefore $7.90 - 4.18 = 3.72$. The profit/loss, ignoring the impact of discounting, is displayed below.

Stock Price Range	*Long \$25 call*	*Short \$30 call*	*Total payoff*	*Profit/Loss*
$S_T \le 25$	$\max(S_T - 25,0) = 0$	$-\max(S_T - 30,0) = 0$	0	$0 - 3.72 = -3.72$
$25 < S_T < 30$	$\max(S_T - 25,0) = S_T - 25$	$-\max(S_T - 30,0) = 0$	$S_T - 25$	$S_T - 25 - 3.72 = S_T - 28.72$
$S_T \ge 30$	$\max(S_T - 25,0) = S_T - 25$	$-\max(S_T - 30,0) = 30 - S_T$	30 - 25	$30 - 25 - 3.72 = 1.28$

0.2 0.3 0.3

0.1



0.1 1 0.1

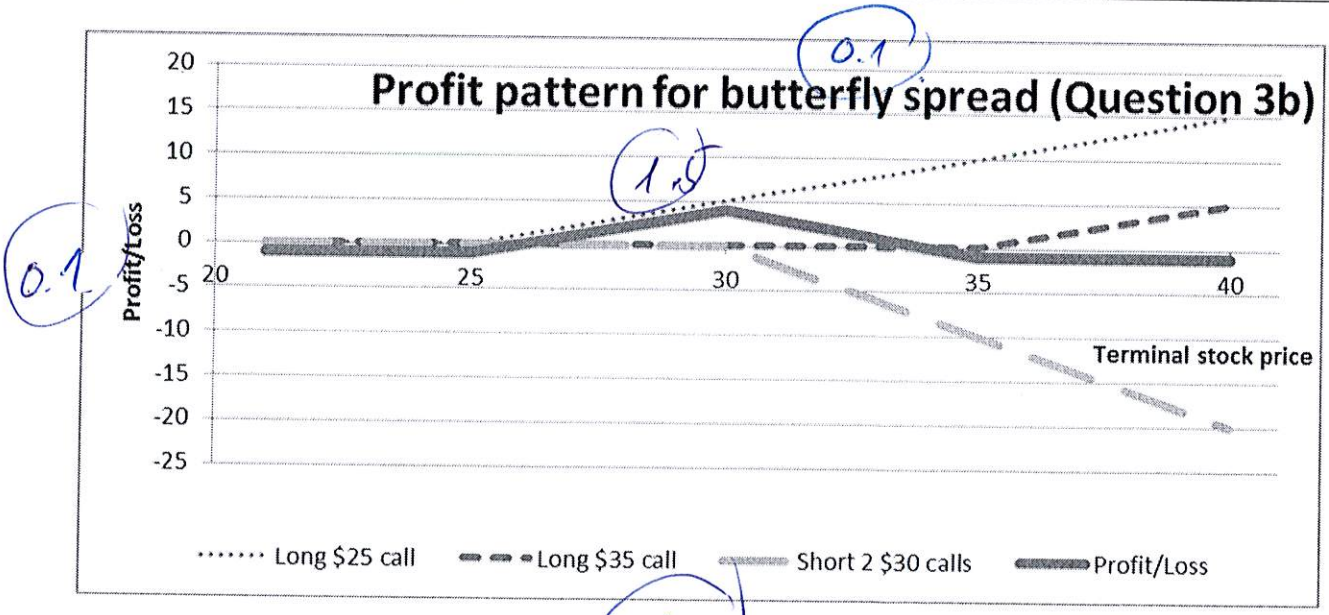
0.5

The investor is engaging in this strategy hoping that the stock price will increase in one year.

b. *Put options with strike prices of \$25, \$30, and \$35 cost \$0.70, \$2.14 and \$4.57, respectively. All these options have maturity of one year. Demonstrate the profit/ loss pattern for a trading strategy (called butterfly spread) of buying one \$25 put option, buying one \$35 put option and selling 2 \$30 put options. Why would an investor be engaging such a strategy? [3 pts]*

Answer: Put options with maturities of one year and strike prices of 25, 30, and 35 cost \$0.70, \$2.14, and \$4.57, respectively. The cost of the butterfly spread is \$1.00 (allowing for rounding errors) since $0.70+4.57-2\times2.14=0.99$. The profits, ignoring the impact of discounting, are given below.

Stock Price Range	*Long 1 \$25 call*	*Long 1 \$35 call*	*Short 2 \$30calls*	*Total payoff*	*Profit/ Loss*
$S_T \le 25$	$\max(S_T - 25,0) = 0$	$\max(S_T - 35,0) = 0$	$-2\max(S_T - 30,0) = 0$	0	−1.00
$25 < S_T < 30$	$\max(S_T - 25,0) = S_T - 25$	$\max(S_T - 35,0) = 0$	$-2\max(S_T - 30,0) = 0$	$S_T - 25$	$S_T - 26.00$
$30 \le S_T < 35$	$\max(S_T - 25,0) = S_T - 25$	$\max(S_T - 35,0) = 0$	$-2\max(S_T - 30,0) = -2(S_T - 30)$	$S_T - 25 - 2S_T + 60 = 35 - S_T$	$34.00 - S_T$
$S_T \ge 35$	$\max(S_T - 25,0) = S_T - 25$	$\max(S_T - 35,0) = S_T - 35$	$-2\max(S_T - 30,0) = -2(S_T - 30)$	$S_T - 25 + S_T - 35 - 2S_T + 60 = 0$	-1.00

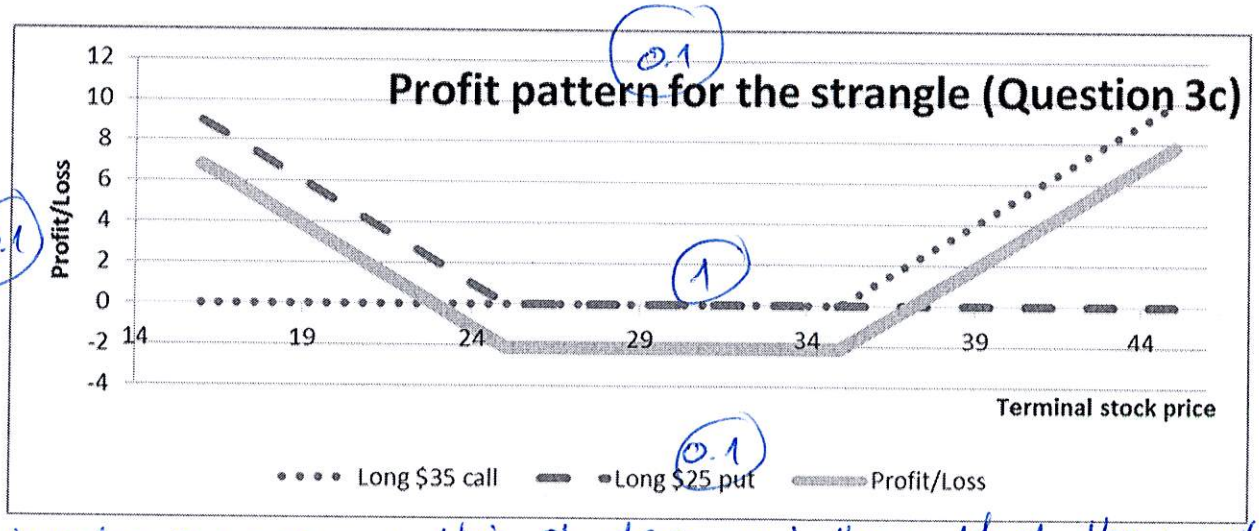


An investor engaging in this strategy thinks that the stock price will stay close to \$30 in the year.

c. A six-month call option with a strike price of \$35 costs \$1.85. A six-month put option with a strike price of \$25 costs \$0.28. Demonstrate the profit/loss pattern for a trading strategy (called strangle) of buying the call and put options. Why would an investor be engaging such a strategy? [2.5 pts]

Answer: A six-month call option with a strike price of 35 costs 1.85. A six-month put option with a strike price of 25 costs 0.28. The cost of the strangle is therefore $1.85+0.28=2.13$. The profits ignoring the impact of discounting are

Stock Price Range	*Long \$35 call*	*Long \$25 put*	*Total Payoff*	*Profit/Loss*
$S_T \le 25$	$\max(S_T - 35,0) = 0$	$\max(25 - S_T, 0) = 25 - S_T$	$25 - S_T$	$22.87 - S_T$
$25 < S_T < 35$	$\max(S_T - 35,0) = 0$	$\max(25 - S_T, 0) = 0$	0	−2.13
$S_T \ge 35$	$\max(S_T - 35,0) = S_T - 35$	$\max(25 - S_T, 0) = 0$	$S_T - 35$	$S_T - 37.13$



An investor engaging in this strategy is betting that there will be a large price move in one year but is uncertain whether it will be an increase or decrease.

♣Required Assignment Question 4 (Cvitanic and Zapatero, problem 18 of chapter 1) [4 points]

At time zero you enter into a short position in futures contract on 20 shares of stock XYZ at the futures price of \$50.00. Moreover, you write (sell) 5 "exotic" options of the following type: they are put options, but using as the underlying asset the average of today's stock price and the stock price at maturity of the underlying asset. The option's strike price is \$52, the option selling price today is \$5.00 per option and

today's stock price is \$49.00 per share. The maturity of all your positions is T = months. What is your total profit or loss two months from now if

(a) at maturity the price of one stock share is \$57.00? [2 pts]

Answer: You lose 20(57-50) = \$140 in the futures contract. You receive 5(5) = \$25.00 for the options. Since the average of the initial and the final stock price is \$53, the put option is not exercised, that is, you have to pay $(5)(52-53)^+ = 0$ as the options' payoff. Your total profit or loss is

$$P\&L = -20(57-50) + 5(5) - 0 = -\$115, \text{ which is a loss.}$$

(b) at maturity the price of one stock share is \$47.00? [2 pts]

Answer: You gain 20(50-47) = \$60 from the futures contract. You receive 5(5) = \$25.00 for the options. The average of the initial and the final stock price is \$48, the put option is exercised, and you have to pay $5(52-48)^+ = \$20$ as the options' payoff. Your total profit or loss is

$$P\&L = 20(50-47) + 5(5) - 5(52-48)^+ = \$65, \text{ which is a gain.}$$