# Statistical Sciences 3520B <br> Assignment No. 2 <br> Due Date: 13:30, 12 March 2014 <br> WSC 240 

N.B. Your assignment paper must include the Marking Scheme sheet as a cover page. This can be downloaded from the course website. Failure to follow this instruction will result to a 2-point deduction on your assignment mark. Additionally, each instruction (if any) not correctly followed in the marking scheme sheet will also lead to a 0.2-point deduction.

NOTE: Only problems marked with $\bullet$ are required for submission; there are 4 of them and they are found on pages 4 and 5 . The other problems are intended as additional practice exercises.

## IMPORTANT CONSIDERATIONS WHEN REQUESTING FOR RE-MARKING OF ASSIGNMENT OR MIDTERM

 EXAM PAPERS: If there is any compelling reason for you to believe that your assignment or exam paper was marked incorrectly by the grader, you can request for a re-marking by writing your concerns on a separate sheet. Attach this sheet to the assignment or exam that you would like to be re-marked. Explain very carefully and diplomatically your arguments why you think there is a mistake in the marking. Do not scribble on the assignment or exam paper, otherwise the grader (TA) may think that you changed or altered your answer(s). You should be aware that when you request for re-marking the grader (TA) may re-assess the entire paper and not just the item(s) you queried about. As a result, your mark may increase but there is also a risk that it may decrease or it could remain the same. Remember, you only have one opportunity to make an appeal. So, make sure your request is clear and complete.
## GUIDELINES ON SUBMITTING ASSIGNMENTS

- Do not submit your rough work! Do the problem set and then re-write it at least once - neatly, with adequate amount of clear explanation. The rewriting stage is the most important one for finding errors in one's work, and it will also deepen
your understanding of the subject matter. Assignments are marked for both technical correctness and elegance of presentation.
- Bear in mind to include a sufficient amount of explanation about your work so that any marker does not have to guess what you mean. The grader of your work will determine if you understand what you are writing, not merely that you reach the particular correct answer.
- On questions where a computer output is required, include the output in the text of your answer at the appropriate locations - do not put it all in a bunch at the end of your assignment. Unless, you are instructed to submit your work in a CD or disc, you are expected to hand in a PRINTED COPY. Assignments sent via e-mail will not be accepted.
- YOU MUST WRITE YOUR OWN WORK IN YOUR OWN WORDS, using full sentences and proper English grammar. It is your responsibility to familiarise yourself with the provisions of the University Regulation concerning academic integrity and honesty. Any behaviour that can potentially lead to plagiarism and cheating is a serious offence and carries with it severe penalty.

For problems not required for submission, you will learn and benefit more if you attempt solving them first before looking at their solutions.

## BINOMIAL OPTION PRICING PROBLEMS

## Problem 12.2

Explain the no-arbitrage and risk-neutral valuation approaches to valuing a European option using a one-step binomial tree.

## Problem 12.3

What is meant by the delta of a stock option?

## Problem 12.5

A stock price is currently $\$ 100$. Over each of the next two six-month periods it is expected to go up by $10 \%$ or down by $10 \%$. The risk-free interest rate is $8 \%$ per annum with continuous compounding. What is the value of a one-year European call option with a strike price of $\$ 100$ ?

## Problem 12.6

For the situation considered in Problem 12.5, what is the value of a one-year European put option with a strike price of $\$ 100$ ? Verify that the European call and European put prices satisfy put-call parity.

## Problem 12.8

Consider the situation in which stock price movements during the life of a European option are governed by a two-step binomial tree. Explain why it is not possible to set up a position in the stock and the option that remains riskless for the whole of the life of the option.

## Problem 12.10

A stock price is currently $\$ 80$. It is known that at the end of four months it will be either $\$ 75$ or $\$ 85$. The risk-free interest rate is $5 \%$ per annum with continuous compounding. What is the value of a four-month European put option with a strike price of $\$ 80$ ? Use no-arbitrage arguments.

## Problem 12.11

A stock price is currently $\$ 40$. It is known that at the end of three months it will be either $\$ 45$ or $\$ 35$. The risk-free rate of interest with quarterly compounding is $8 \%$ per annum. Calculate the value of a three-month European put option on the stock with an exercise price of $\$ 40$. Verify that no-arbitrage arguments and risk-neutral valuation arguments give the same answers.

## Problem 12.14

A stock price is currently $\$ 25$. It is known that at the end of two months it will be either $\$ 23$ or $\$ 27$. The risk-free interest rate is $10 \%$ per annum with continuous compounding. Suppose $S_{T}$ is the stock price at the end of two months. What is the value of a derivative that pays off $S_{T}^{2}$ at this time?

## Problem 12.15

Calculate $u, d$, and $p$ when a binomial tree is constructed to value an option on a foreign currency. The tree step size is one month, the domestic interest rate is $5 \%$ per annum, the foreign interest rate is $8 \%$ per annum, and the volatility is $12 \%$ per annum.

## \&REQUIRED PROBLEM \#1 [4 points]

## Additional Problem 1

A stock price is currently $\$ 25$. It is known that at the end of 4 months it will be either $\$ 30$ or $\$ 21$. The risk-free rate of interest with continuous compounding is $12 \%$ per annum. Calculate the value of a 4 -month European call option with an exercise price of $\$ 24$. Verify that no-arbitrage arguments and risk-neutral valuation arguments give the same answer.

## \&REQUIRED PROBLEM \#2 [5 points]

## Additional Problem 2

In a two-period binomial model with $r=1 \%$ per period, the current stock price is $\$ 100$, and $u=1.02$ and $d=0.98$. Consider an option that expires after two periods, and pays the value of the squared stock price, $S\left(t^{2}\right.$, if the stock price $S(t)$ is higher than $\$ 100$ when the option is exercised. Otherwise (when $S(t)$ is less than or equal to $\$ 100$ ), the option pays zero. If the option under consideration is an American-type, find its price.

## VALUE-AT-RISK PROBLEMS

## Problem 21.1

Consider a position consisting of a $\$ 100,000$ investment in asset A and a $\$ 100,000$ investment in asset B. Assume that the daily volatilities of both assets are $1 \%$ and that the coefficient of correlation between their returns is 0.3 . What is the 5 -day $99 \%$ VaR for the portfolio?

## Problem 21.6

Suppose a company has a portfolio consisting of positions in stocks, bonds, foreign exchange, and commodities. Assume there are no derivatives. Explain the assumptions underlying (a) the linear model and (b) the historical simulation model for calculating VaR.

## 』REQUIRED PROBLEM \#3 [4 points]

## Additional Problem 3

Brian Griffin is a small investor who has $70 \%$ of his portfolio invested in a marketindex fund, and $30 \%$ in a small-stocks fund. The mean monthly return rate of the market-index fund is $1.5 \%$ with a standard deviation of $0.9 \%$. The small-stocks fund has the mean monthly return rate of $2.2 \%$ with standard deviation of $1.2 \%$. The correlation between the two funds is 0.13 . Assume normal distribution for the
return rates. What is the monthly VaR at $99 \%$ level for Brian's portfolio if the portfolio value today is $\$ 100,000$ ?

## BASIC ELEMENTS OF STOCHASTIC PROCESSES <br> *REQUIRED PROBLEM \#4 [7 points]

## Additional Problem 4

Consider the experiment of drawing (with replacement) a ball three times from a bag containing a blue ball (B) and a red ball (R).
(a) Write down the set or sample space, $\Omega$, containing all possible outcomes. If we consider the collection of all subsets of $\Omega$, how many sets are there in this collection? [1 pt]
(b) Suppose the probability of getting B is $3 / 7$ and the probability of getting R is $4 / 7$. Define or construct the probability measure for each individual element $\omega \in \Omega$. [1 pt]
(c) Write down the $\sigma$-algebra or $\sigma$-field $\left\{\mathfrak{I}_{i}\right\}$ keeping track the outcomes of each drawing for $i=0,1,2,3$. [2 pts]
(d) Let $\Omega$ be given as in (a) and consider the binomial asset pricing model where $S_{0}=20, d=4 / 5$ and $u=5 / 4$ so that $\left\{S_{i}\right\}$ is a stochastic process, i.e., $S_{i}$ 's are random variables for $i=0,1,2,3$. Find $S_{2}(\varpi)$, i.e., what is the function $S_{2}(\varpi)$ ? $[1 \mathrm{pt}]$
(e) Consider the interval $\left[\mathrm{e}^{\pi}-9, \pi^{\mathrm{e}}+9\right]$. What is the pre-image under $S_{2}$ of this interval? Recall that $S_{2}$ is a random variable and by definition it maps $\Omega$ into $\Re$. [2 pts]

