Statistical Sciences 3520B
Winter 2014
Solutions to Assignment No. 3 Practice Problems

♣**Required Assignment Problem #1 [8 points]**
Solution is given in a separate sheet.

♣**Required Assignment Problem #2 [4 points]**
Solution is given in a separate sheet.

♣**Required Assignment Problem #3 [8 points]**
Solution is given in a separate sheet.

Additional Problems (not required for submission)
Question 1
Consider the modelling framework of the 3-asset market. Suppose we have a market with a risky asset having pay-off [1 2 3], a cash with pay-off [1 1 1] and a security (which could be a derivative) with pay-off [1 1 0]. Suppose the risk-neutral probabilities of attaining states 1, 2 and 3 are q_1, q_2 and q_3, respectively. Is the inverse problem of determining the risk-neutral probability measure (q_1 q_2 q_3) solvable? Why or why not? Assume that you are given a risk-free rate r, which is a positive constant.

SOLUTION:
Let S be the current price of the risky asset and C be the current value of the derivative. If the cash with pay-off 1 is to be received in the future (time 1) then its present value is 1/(1+r). Thus, the system to be solved would be

$$S(1+r) = 1q_1 + 2q_2 + 3q_3$$
$$\frac{1}{1+r}(1+r) = 1 = (1q_1 + 1q_2 + 1q_3)$$
$$C(1+r) = 1q_1 + 1q_2 + 0q_3$$

The inverse problem is to solve for q_1, q_2 and q_3, given S, r and C. The system has solution $q_1 = 3-(1+r)(S+C)$, $q_2 = (1+r)(S+2C)-3$ and $q_3 = 1-(1+r)C$. Hence, since the risk-neutral probabilities were recovered from market prices (S and C), the inverse problem is solvable in this case.

Question 2

Consider a single-period CRR model with interest rate 0.05, $S(0)=10$, $u=1.2$ and $d=0.98$. Suppose you have written an option that pays the value of the square root of the absolute value of the difference between the stock price at maturity and \$10.00; that is, it pays $\sqrt{|S(1)-10|}$. How many shares of the stock should you buy to replicate this pay-off? What is the cost of the replicating portfolio?

SOLUTION:

Let α and Δ be the respective holdings in the money market account and stock investments.

We need to have
$$\begin{aligned} 1.05\alpha + 12\Delta &= \sqrt{|12-10|} = 1.4142 \\ 1.05\alpha + 9.8\Delta &= \sqrt{|9.8-10|} = 0.4472 \end{aligned}$$

Solving this system we get $\alpha = -3.6765$ and $\Delta = 0.4395$.

Thus, we need to buy 0.4395 shares and the cost of the replicating portfolio is $\alpha + 10\Delta = 0.7189$.

PROBLEMS INVOLVING SWAPS

Problem 7.1

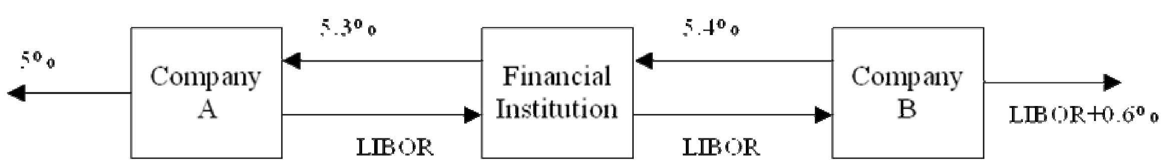
Companies A and B have been offered the following rates per annum on a $20 million five-year loan:

	Fixed Rate	*Floating Rate*
Company A	5.0%	LIBOR+0.1%
Company B	6.4%	LIBOR+0.6%

Company A requires a floating-rate loan; company B requires a fixed-rate loan. Design a swap that will net a bank, acting as intermediary, 0.1% per annum and that will appear equally attractive to both companies.

SOLUTION:

A has an apparent comparative advantage in fixed-rate markets but wants to borrow floating. B has an apparent comparative advantage in floating-rate markets but wants to borrow fixed. This provides the basis for the swap. There is a 1.4% per annum differential between the fixed rates offered to the two companies and a 0.5% per annum differential between the floating rates offered to the two companies. The total gain to all parties from the swap is therefore $1.4-0.5=0.9\%$ per annum. Because the bank gets 0.1% per annum of this gain, the swap should make each of A and B 0.4% per annum better off. This means that it should lead to A borrowing at LIBOR -0.3% and to B borrowing at 6.0%. The appropriate arrangement is therefore shown below.



Swap design for Problem 7.1

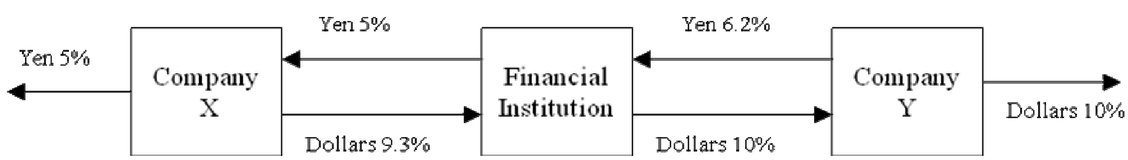
Problem 7.2

Company X wishes to borrow U.S. dollars at a fixed rate of interest. Company Y wishes to borrow Japanese yen at a fixed rate of interest. The amounts required by the two companies are roughly the same at the current exchange rate. The companies have been quoted the following interest rates, which have been adjusted for the impact of taxes:

	Yen	*Dollars*
Company X	5.0%	9.6%
Company Y	6.5%	10.0%

Design a swap that will net a bank, acting as intermediary, 50 basis points per annum. Make the swap equally attractive to the two companies and ensure that all foreign exchange risk is assumed by the bank.

SOLUTION:
X has a comparative advantage in yen markets but wants to borrow dollars. Y has a comparative advantage in dollar markets but wants to borrow yen. This provides the basis for the swap. There is a 1.5% per annum differential between the yen rates and a 0.4% per annum differential between the dollar rates. The total gain to all parties from the swap is therefore $1.5-0.4=1.1\%$ per annum. The bank requires 0.5% per annum, leaving 0.3% per annum for each of X and Y. The swap should lead to X borrowing dollars at $9.6-0.3=9.3\%$ per annum and to Y borrowing yen at $6.5-0.3=6.2\%$ per annum. The appropriate arrangement is therefore shown below. All foreign exchange risk is borne by the bank.



Swap design for Problem 7.2

Problem 7.6
Explain the difference between the credit risk and the market risk in a financial contract.

SOLUTION:
Credit risk arises from the possibility of a default by the counterparty. Market risk arises from movements in market variables such as interest rates and exchange rates. A complication is that the credit risk in a swap is contingent on the values of market variables. A company's position in a swap has credit risk only when the value of the swap to the company is positive.

Problem 7.8

Explain why a bank is subject to credit risk when it enters into two offsetting swap contracts.

SOLUTION:

At the start of the swap, both contracts have a value of approximately zero. As time passes, it is likely that the swap values will change, so that one swap has a positive value to the bank and the other has a negative value to the bank. If the counterparty on the other side of the positive-value swap defaults, the bank still has to honor its contract with the other counterparty. It is liable to lose an amount equal to the positive value of the swap.

Problem 7.15

Why is the expected loss from a default on a swap less than the expected loss from the default on a loan with the same principal?

SOLUTION:

In an interest-rate swap a financial institution's exposure depends on the difference between a fixed-rate of interest and a floating-rate of interest. It has no exposure to the notional principal. In a loan the whole principal can be lost.

Problem 7.16

A bank finds that its assets are not matched with its liabilities. It is taking floating-rate deposits and making fixed-rate loans. How can swaps be used to offset the risk?

SOLUTION:

The bank is paying a floating-rate on the deposits and receiving a fixed-rate on the loans. It can offset its risk by entering into interest rate swaps (with other financial institutions or corporations) in which it contracts to pay fixed and receive floating.