# Statistical Sciences 3520B <br> Assignment No. 3 <br> Due Date: 14:20, 04 April 2014 

Submission of Assignment 3: Put your assignment in the drop box, labelled SS 3520B, across WSC 240. The assigned TA will monitor the arrival of assignments, and as per the course outline UNDER NO CIRCUMSTANCES WILL LATE ASSIGNMENTS BE ACCEPTED.

> Return of Marked Assignment 3: An e-mail will be sent to inform you when and where the marked assignment 3 papers can be picked up. The solutions of the required questions in this assignment will be posted in the course website by 07 April 2014. You have one week after the e-mail is sent to make a re-marking request of the last assignment. Please ask a dep't staff to put your re-marking note/request in the instructor's mailbox.

For your peace of mind, the final exam papers will be graded by the instructor. No TAs will be involved in the marking.

NOTE: Only problems marked with are required for submission. The other problems are intended as additional practice exercises.

## IMPORTANT CONSIDERATIONS WHEN REQUESTING FOR RE-MARKING OF ASSIGNMENT OR MIDTERM

 EXAM PAPERS: If there is any compelling reason for you to believe that your assignment or exam paper was marked incorrectly by the grader, you can request for a re-marking by writing your concerns on a separate sheet. Attach this sheet to the assignment or exam that you would like to be re-marked. Explain very carefully and diplomatically your arguments why you think there is a mistake in the marking. Do not scribble on the assignment or exam paper, otherwise the grader (TA) may think that you changed or altered your answer(s). You should be aware that when you request for re-marking the grader (TA) may re-assess the entire paper and not just the item(s) you queried about. As a result, your mark may increase but there is also a risk that it may decrease or it could remain the same. Remember, you only have one opportunity to make an appeal. So, make sure your request is clear and complete.
## GUIDELINES ON SUBMITTING ASSIGNMENTS

- Do not submit your rough work! Do the problem set and then re-write it at least once - neatly, with adequate amount of clear explanation. The rewriting stage is the most important one for finding errors in one's work, and it will also deepen your understanding of the subject matter. Assignments are marked for both technical correctness and elegance of presentation.
- Bear in mind to include a sufficient amount of explanation about your work so that any marker does not have to guess what you mean. The grader of your work will determine if you understand what you are writing, not merely that you reach the particular correct answer.
- On questions where a computer output is required, include the output in the text of your answer at the appropriate locations - do not put it all in a bunch at the end of your assignment. Unless, you are instructed to submit your work in a CD or disc, you are expected to hand in a PRINTED COPY. Assignments sent via e-mail will not be accepted.
- YOU MUST WRITE YOUR OWN WORK IN YOUR OWN WORDS, using full sentences and proper English grammar. It is your responsibility to familiarise
yourself with the provisions of the University Regulation concerning academic integrity and honesty. Any behaviour that can potentially lead to plagiarism and cheating is a serious offence and carries with it severe penalty.

For problems not required for submission, you will learn and benefit more if you attempt solving them first before looking at their solutions.

## \& Required Assignment Problem \#1 [8 points]

In the lecture it was pointed out that the continuous-time limit of the binomial model for the stock price is the geometric Brownian motion given by

$$
S_{T}=S_{0} \exp \left[\left(\mu-\sigma^{2} / 2\right) T+\sigma W_{T}\right]
$$

where $S_{T}$ is the stock price at time $T, S_{0}$ is the stock price at time 0 (assumed to be known), $\mu$ is the stock's expected return and $\sigma$ is the stock price volatility. Here, $W_{T}$ is a stochastic process such that $W_{T} \sim N(0, T)$.

Assume that the stock price is currently $\$ 50$. Assume that the expected return from the stock is $18 \%$ per annum with continuous compounding and its volatility is $30 \%$ per annum.
(a) What is the probability distribution for the stock price in two years? [2 pts]
(b) Calculate the mean and standard deviation of the distribution. [1 pt]
(c) Determine the $95 \%$ confidence interval. [2 pts]
(d) If a 6-month European call option is written on this stock with a strike price of $X=50$, what is the risk-neutral probability that the option will be exercised? Assume a risk-free rate of $10 \%$ per annum with continuous compounding. [ 3 pts ]

## ^Required Assignment Problem \#2 [4 points]

Assume a single-period model with $r=0.05$ and $\mathrm{S}(0)=100$. After one period, the stock price can either go up to 101 or down to 99 .
(a) Find a replicating portfolio for a put option with the pay off function $g(s)=$ $(100-\mathrm{s})^{+}=\max (100-\mathrm{s}, 0)$. Use a replicating portfolio consisting of $\boldsymbol{\alpha}$ amount held in a savings account and $\Delta$ shares of holdings in stock. [3 pts] [Note: You will get a zero for this question if you construct a different replicating porffolio.]
(b) Using your replicating portfolio in (a), what is the price of the put option in question? [ 1 pt ]
\&Required Assignment Problem \#3 [8 points]
Consider a financial market with 3 states of the world.
(a) Consider a portfolio consisting of: (i) a risky asset whose pay-off is given by the vector [ 369 ] and (ii) cash (a riskless asset) whose pay-off is given by the vector [3 3 3]. Is it possible to purchase, or at least create (i.e., make available) a derivative whose pay-off is [ 3000 ]? Justify your answer. [ 3 pts ]

In light of your answer to the first question, is the market being considered here with the two basic securities complete or incomplete? Why? [1 pt]
(b) Suppose that in addition the two assets in the market considered in (a), a security with pay-off [ $\left.\begin{array}{lll}3 & 3 & 0\end{array}\right]$ is introduced; thus a "new" or "revised" market is formed. Under this "revised" market, describe how will you obtain the pay-offs $\left[\begin{array}{lll}1 & 0 & 0\end{array}\right]$, $\left[\begin{array}{lll}0 & 1 & 0\end{array}\right]$ and $\left[\begin{array}{lll}0 & 0 & 1\end{array}\right]$, which are called state-contingent claims. [3 pts]

Is the claim $\left[\begin{array}{lll}3 & 0 & 0\end{array}\right]$ attainable under this 3-asset market? Why? [1 pt $]$

## Additional Problems (not required for submission) Question 1

Consider again the modelling framework of the 3-asset market in Required Assignment Problem \#3. Suppose we have a market with a risky asset having payoff $\left[\begin{array}{lll}1 & 2 & 3\end{array}\right]$, a cash with pay-off $\left[\begin{array}{lll}1 & 1 & 1\end{array}\right]$ and a security (which could be a derivative) with pay-off $\left[\begin{array}{lll}1 & 1 & 0\end{array}\right]$. Suppose the risk-neutral probabilities of attaining states 1,2 and 3 are $q_{1}, q_{2}$ and $q_{3}$, respectively. Is the inverse problem of determining the riskneutral probability measure $\left(\begin{array}{lll}q_{1} & q_{2} & q_{3}\end{array}\right)$ solvable? Why or why not? Assume that you are given a risk-free rate $r$, which is a positive constant.

## Question 2

Consider a single-period CRR model with interest rate $0.05, S(0)=10, u=1.2$ and $d=0.98$. Suppose you have written an option that pays the value of the square root of the absolute value of the difference between the stock price at maturity and $\$ 10.00$; that is, it pays $\sqrt{|S(1)-10|}$. How many shares of the stock should you buy to replicate this pay-off? What is the cost of the replicating portfolio?

## PROBLEMS INVOLVING SWAPS

## Problem 7.1

Companies A and B have been offered the following rates per annum on a $\$ 20$ million five-year loan:

|  | Fixed Rate | Floating Rate |
| :--- | :--- | :--- |
| Company A | $5.0 \%$ | LIBOR $+0.1 \%$ |
| Company B | $6.4 \%$ | LIBOR $+0.6 \%$ |

Company A requires a floating-rate loan; company B requires a fixed-rate loan.
Design a swap that will net a bank, acting as intermediary, $0.1 \%$ per annum and that will appear equally attractive to both companies.

## Problem 7.2

Company X wishes to borrow U.S. dollars at a fixed rate of interest. Company Y wishes to borrow Japanese yen at a fixed rate of interest. The amounts required by the two companies are roughly the same at the current exchange rate. The companies have been quoted the following interest rates, which have been adjusted for the impact of taxes:

|  | Yen | Dollars |
| :--- | :---: | :---: |
| Company X | $5.0 \%$ | $9.6 \%$ |
| Company Y | $6.5 \%$ | $10.0 \%$ |

Design a swap that will net a bank, acting as intermediary, 50 basis points per annum. Make the swap equally attractive to the two companies and ensure that all foreign exchange risk is assumed by the bank.

## Problem 7.6

Explain the difference between the credit risk and the market risk in a financial contract.

## Problem 7.8

Explain why a bank is subject to credit risk when it enters into two offsetting swap contracts.

## Problem 7.15

Why is the expected loss from a default on a swap less than the expected loss from the default on a loan with the same principal?

## Problem 7.16

A bank finds that its assets are not matched with its liabilities. It is taking floatingrate deposits and making fixed-rate loans. How can swaps be used to offset the risk?

