# Stats 3520b – Week of 24–28 February 2014

## SUMMARY OF IMPORTANT POINTS DISCUSSED IN THE LECTURE

The following concepts were covered/reviewed:

1. It is assumed that estimates of daily volatilities and correlations are available all throughout the discussion of VaR calculation. The estimation of volatilities are covered under the topics of EWMA/ARCH/GARCH (typically expected to be discussed in SS 4521G and also usually taken in econometrics-focused or statistical analysis of financial data courses).

2. Examples were given in the lecture to demonstrate how to calculate VaR within the framework of a linear model for a portfolio with one and two (possibly correlated) assets. The discussion was extended to the calculation of VaR metric when a portfolio contains $n$ correlated assets. The benefit of portfolio diversification was also pointed out in the context of VaR computation.

3. If the portfolio contains options, the change in the portfolio value denoted by $\delta P$ is not linearly related to the percentage changes in market variables. It is necessary to take into account the quadratic relationship between $\delta P$ and the percentage changes in market variables; this gives a more accurate approximation than the one given by a linear model. The quadratic extension will **not** be included in the exam as this requires materials covered in SS 4521G. However, anyone interested to learn this particular topic can consult the book of Hull (2012).

4. When the portfolio contains bonds, VaR calculation will involve *cash-flow mapping*. Hull (2012) provides a straightforward discussion of

cashflow mapping. This will **not** be covered in the exam.

5. The discussion of VaR culminated by looking at back testing. Back testing involves testing how the VaR estimates would have performed in the past. Assuming that we are computing a 1-day 99% VaR, back testing will involve examining how often the loss in a day exceeded the 1-day 99% VaR calculated for that day. If this happened on around 1% of the days, one can feel confident about the methodology used in the VaR calculation. But if this happened, say, 4% of the days, the methodology needs to be investigated.

6. The concept of stress testing together with the application and importance of principal component analysis in VaR calculation was also discussed. Stress testing is the examination of the impact of extreme market moves on the value of the portfolio.

7. A comparison between the historical approach and model-building approach, with the advantages and disadvantages of each approach, was presented.

**BASIC ELEMENTS OF STOCHASTIC PROCESSES RELEVANT TO DERIVATIVE PRICING**

8. A **probability space** is a triplet $(\Omega, \mathcal{F}, P)$, where
   $\Omega$ is the set of all possible outcomes of a random experiment;
   $\mathcal{F}$ is a $\sigma$**-field** or $\sigma-$**algebra**, i.e., it is a non-empty collection of subsets of $\Omega$ and it is closed under set complementation and countable union; and
   $P$ is a **probability measure**, i.e., $P : \mathcal{F} \to [0, 1]$ and $P$ is countably additive.

9. A **random variable** (RV) $X$ is a function where $X : \Omega \to \mathbb{R}$. More formally, $X$ is a RV if $X^{-1}(B) \in \mathcal{F}$ for some Borel set $B$. A **Borel set** is any set that can be formed from open sets (or, equivalently, from closed sets) through the operations of countable union, countable intersection, and relative complement.

10. A **stochastic process** $X_t(\omega)$ or $X(t, \omega)$ is a family of RVs, where $\omega \in \Omega$ and $t$ is a time index that could either be discrete or continuous.

11. The concept of probability measure for a discrete RV was also discussed. Intuitively, this refers to a set of probabilities.