Stats 3520b – Week of 03–07 February 2014

SUMMARY OF IMPORTANT POINTS DISCUSSED IN THE LECTURE

The following concepts were covered/reviewed:

1. Pricing European options using binomial trees under a one-period framework:
   Let S =current stock price, u =“appreciation” factor when the stock price moves up and d =“depreciation” factor when the stock price moves down and so
   $u-1$ =proportional increase when there is an up movement. The stock price goes up to the new level Su ($u > 1$).
   $1-d$ =proportional decrease when there is a down movement. The stock price goes down to the new level Sd ($d < 1$).
   r =risk-free rate.

   By considering a riskless portfolio (consisting of a long position in Δ shares and a short position in one option), it was shown that $\Delta = \dfrac{f_u - f_d}{Su - Sd}$ where f_u =pay-off from the option when the stock price is Su and f_d =pay-off from the option when the stock price is Sd. Finally, we equate the cost of setting up the portfolio with its present value. If f denotes the price of an option then

   $$f = e^{-rT}\left[qf_u + (1-q)f_d\right] \qquad (1)$$

   where $q = \dfrac{e^{rT} - d}{u - d}$.

   The above argument can be extended to a two-step binomial pricing model, and in general to an $n-$step binomial pricing. As we increase n, the option pricing formula will converge to the Black-Scholes option pricing representation.

2. Pricing European options using binomial trees under a two-period framework:
   For a two-step binomial pricing model, let
   f_u=pay-off from the option when the stock price is Su
   f_{uu}= pay-off from the option when the stock price is Su^2
   f_{ud}=pay-off from the option when the stock price is $Sud = Sdu$
   f_d=pay-off from the option when the stock price is Sd
   f_{dd}=pay-off from the option when the stock price is Sd^2

   Over one period, the option pricing formula is $f = e^{-rT}\left[qf_u + (1-q)f_d\right]$.
   For a two-period model in which the length of one period is δT the price of a European option is $f = e^{-2r(\delta T)}\left[q^2 f_{uu} + 2q(1-q)f_{ud} + (1-q)^2 f_{dd}\right]$.

3. If we generalise the use of binomial trees by adding more steps to the tree, we find that the risk-neutral valuation principle continues to hold. That is, the option price is always equal to its expected pay-off in a risk-neutral world discounted at the risk-free rate; this is, assuming of course, that the interest rate is constant throughout the life of the option contract.

4. Procedures in pricing of American options using binomial trees:
   (i) Work backwards through the tree from the end to the beginning, testing at each node to see whether early exercise is optimal.
   (ii) The value of the option at the final nodes is the same as for the European option.
   (iii) At earlier nodes the value of the option is the maximum of
   (a) $f = e^{-r(\delta T)}\left[qf_u + (1-q)f_d\right]$, where δT is the length of one-period time interval
   and
   (b) the pay-off from early exercise.

   The above procedure was illustrated using a numerical example in the lecture. It is apparent that if early exercise is optimal, the price of the American call/put is greater than the price of the corresponding

European call/put.