Stats 3520b – Week of 03–07 March 2014

SUMMARY OF IMPORTANT POINTS DISCUSSED IN THE LECTURE

The following concepts were covered/reviewed:

1. We continued the discussion of **basic elements of stochastic processes relevant to derivative pricing**.

2. The filtration process. A **filtration** represents the history of a price process. More formally, a filtration is a sequence of non-decreasing sub-$\sigma-$fields or sub-$\sigma-$algebras (or collection of subsets of Ω). Some examples were given in the lecture to illustrate the concept of $\sigma-$field and filtration.

3. ***All models that are considered in finance are assumed to be defined on a probability space equipped with a filtration.***

4. A **contingent claim** on the binomial tree is a function of the nodes at a claim-time horizon T. Note that this is also a function of the filtration $\{\mathcal{F}_T\}$. Claims can either be (sample) path-dependent or path-independent.

5. **Conditional expectation** operator $E^P[\ \cdot\ |\mathcal{F}_k]$: This extends the idea of expectation to two parameters, namely a measure P and a history up to time k. This is an expectation along the latter portion of paths which have initial segment $\{\mathcal{F}_k\}$; if it makes it easier you may view the node attained at time k as the new root of the binomial tree and take

expectations of future claims from there.

6. A stochastic process S is a **martingale** with respect to a measure Q and a filtration $\{\mathcal{F}_k\}$ if (i) S_k is $\mathcal{F}_k-$adapted, (ii) $E^Q[S_k] < \infty$ and (iii) $E^Q[S_k|\mathcal{F}_j] = S_j, \quad \forall\ j \leq k$. It simply means that the future expected value at time k of the process S under Q conditional on its history until time j is merely the process value at time j.

   An example was given in the lecture.

7. We may occasionally need to use the fact that for $i \leq j$ and claim H,

$$E^P\left[E^P[H|\mathcal{F}_j]|\mathcal{F}_i\right] = E^P[H|\mathcal{F}_i].$$

   In other words, conditioning on the history up to time j and then conditioning on the history up to earlier time i is the same as just conditioning originally up to time i. This result is called the **tower law**.

8. We showed in class that for any claim H, ***the conditional expectation process*** $E^P[H|\mathcal{F}_i]$ ***is always a*** $P-$***martingale using the tower law.***

9. The concepts of probability space, stochastic basis or filtered probability space, $\sigma-$field, and probability measure were defined **formally** in class.

10. Hints and examples were given to help solve Assignment No. 2.