Stats 3520b – Week of 10–14 February 2014

SUMMARY OF IMPORTANT POINTS DISCUSSED IN THE LECTURE

The following concepts were covered/reviewed:

1. In practice, the life of an option is typically divided into at least 30 time steps when implementing the binomial pricing method. This means that 2^{30} (approx 1 billion) possible stock price paths are considered.

2. An important parameter in the pricing and hedging of options is delta (Δ). This parameter represents the number of units of the stock that one should hold for each option shorted in order to create a riskless hedge. More formally, it is the ratio of the change in the price of a stock option to the change in the price of the underlying stock.

3. The construction of a riskless hedge is referred to as *delta hedging*. The delta of a call is positive whilst the delta of a put is negative.

4. It was also demonstrated in the lecture that in order to maintain a riskless hedge using an option and the underlying stock, we need to adjust our holdings in the stock periodically.

5. The complete set of equations for the implementation of the binomial option pricing model is:
   $u = e^{\sigma\sqrt{\delta t}}$; $d = e^{-\sigma\sqrt{\delta t}}$ and $q = \dfrac{e^{r\delta t} - d}{u - d}$,
   where r is the risk-free rate, δt is the time length of one period, and u

and d are the appreciation and depreciation factors, respectively. It is assumed that $u > 1 + r > d$.

6. *The following was briefly mentioned in class, but was not substantially emphasised. You may need this in solving questions in Assignment #2, and will be tested in the Second Midterm.*

   When confronted with problems or questions involving options whose underlying variable is the price of futures contract or foreign exchange rate, all we need to do is modify the above relations in #5.

   Suppose θ is the dividend yield (% of the spot of the underlying asset) provided by the asset underlying the futures contract. Then for options on futures contract we put $\theta = r$. We replace θ by the foreign risk-free rate r_f when the underlying variable is a foreign exchange rate.

   Consequently, the risk-neutral probability q in the implementation of the binomial option pricing approach needs to be modified to

$$q = \frac{e^{(r-\theta)T} - d}{u - d},$$

   where $u - 1$ and $1 - d$ are the respective proportional increase and proportional decrease of the stock price. Note that the change from r to $r - \theta$ hinges on the fact that dividend is an income to the holder of the asset and therefore it decreases the stock price.

   In a similar manner, when employing the risk-neutral probability q in the binomial option pricing model, replace θ by r for options on futures contract and replace θ by the foreign risk-free rate r_f when the underlying variable is a foreign exchange rate.

VALUE-AT-RISK (VaR) CALCULATION

7. VaR is an attempt to provide a single number summarising the total risk in a portfolio of financial assets for senior management; this is used by corporate treasurers and fund managers.

8. In the VaR measure, the goal of a portfolio manager is to make the statement "we are $x\%$ certain that we shall not lose more than V dollars in the next N days."

9. From #8, we note that VaR=$f(N, x)$, i.e., it is a function of the time horizon N and confidence level x. VaR therefore can also be viewed as a loss level in which a manager is $X\%$ sure that it won't be exceeded.

10. One important use of VaR is for a bank whose maintenance capital requirement (as required by the Central Bank or Regulatory Authority) is specified as k times its VaR with $N = 10$ and $x = 99$, where k is a positive real number (usually k is at least 3).

11. To calculate VaR, we examine the probability distribution of changes in portfolio value. The VaR corresponds to the (100-x)th percentile of the distribution of the change in the value of the portfolio over the next N days.

12. VaR is appealing because it is easy to understand. It asks the question "how bad can things get?" and this is essentially the question many managers would like to be answered; it incorporates sensitivity measures of all market variables underlying the portfolio into a single number.

13. Whereas VaR asks the question "how bad can things get?" CVaR (Conditional VaR) asks the question "if things do get bad, how much can we expect to lose?" That is, CVaR is equal to the expected loss in an $N-$day period conditional that we are in the (100-x)% left tail of the distribution.

    For example, if $100 - x = 3$ and $N = 8$, CVaR is the average amount we lose over an 8-day period assuming that a 3%-worse case event will happen.

VaR (not CVaR) is still the popular measure in risk management amongst regulators and senior management.

14. Usually, $N = 1$ is used for time horizon. This is particularly true in practice since there is not enough data to estimate directly the behaviour of the market. However, for an $N-$day VaR, we have the relation $N-$day VaR=(1-day VaR)($\sqrt{N}$).

15. The first approach that was discussed in VaR calculation is historical simulation. This involves the creation of database consisting of the daily movements in all market variables over a period of time. The first simulation trial assumes that the percentage changes in each market variable are the same as those on the first day covered by the database; the second simulation trial assumes that the percentage changes are the same as those on the second day; and so on. The change in the portfolio value, δP, is calculated for each simulation trial, and VaR is computed as the appropriate percentile of the probability distribution of δP.

16. An alternative approach to historical simulation in VaR calculation is model-building. In option valuation, we measure time in years and the volatility of an asset is quoted as a volatility per year. In model-building approach of VaR calculation, we measure time in days and the volatility of an asset is quoted as a volatility per day. We have the relation $\sigma_{\text{day}} = \dfrac{\sigma_{\text{yr}}}{\sqrt{252}}$. For the purpose of calculating VaR, we define <u>the daily volatility of an asset price (or any other variable) equal to the standard deviation of the percentage change in one day</u>.

    Further numerical examples will be given in class.