Stats 3520b – Week of 20–24 January 2014

SUMMARY OF IMPORTANT POINTS DISCUSSED IN THE LECTURE

The following concepts were covered/reviewed:

1. We considered an example (call option on Exxon) of how speculator could use options. The discussion and calculations in the example showed that options like futures provide a form of leverage. For a given investment, the use of options magnifies the financial consequences. Good outcomes become very good, whilst bad outcomes become very bad.

2. The value of a forward contract at the time it is first entered into is zero. At a later time it may prove to have a positive or negative value. The value of a long forward contract, f, in terms of the originally negotiated price K, and the **current** forward price F_0 is given by

$$f = (F_0 - K)e^{-rT}.$$

   To see why the above equation is true, **compare a long forward contract with delivery price F_0 with an otherwise identical long forward contract that has a delivery price of K.** The difference between the two is only in the amount that will be paid for the underlying asset at time T. Under the first forward contract this amount is F_0; under the second contract it is K. The difference of $F_0 - K$ at time T translates to an amount of $(F_0 - K)e^{-rT}$ today.

   Recall again that the value of the contract that has a delivery price of F_0 is by definition zero. It follows that the value of the contract with a delivery price of K is $(F_0 - K)e^{-rT}$.

   The value of a forward contract on a security that provides no income is $f = S_0 - Ke^{-rT}$ since $F_0 = S_0e^{rT}$.

The value of a forward contract on a security that provides a known income with PV I is $f = S_0 - I - Ke^{-rT}$ since $F_0 = (S_0 - I)e^{rT}$.

Lastly, the value of a forward contract on a security that provides a known dividend yield at rate q is $f = S_0e^{-qT} - Ke^{-rT}$ since $F_0 = S_0e^{(r-q)T}$.

N.B.: In each case, the forward price, F, is the value of K which makes f equal zero when the contract was first entered into.

3. An example illustrating how to value a forward contract relying on the valuation formula, $f = (F_0 - K)e^{-rT}$ or $f = S_0 - Ke^{-rT}$, was given in the lecture.

4. A stock index is an indicator that tracks the value of a hypothetical portfolio of stocks. Examples include S&P 500, Nikkei 225, NYSE Composite Index and the MMI.

5. Futures on stock indices are traded. In this case, a stock index can be regarded as the price of a security that pays dividends. The security is the portfolio of stocks underlying the index. If q is the dividend yield rate and S is the current value of the index then the futures price on a stock index is $F = Se^{(r-q)T}$.

   In practice, the dividend yield on the portfolio underlying an index varies week by week throughout the year. The value of q is taken as the average annualised dividend yield during the life of the contract.

6. Stock index futures are used to hedge portfolios of stocks. If $F = Se^{(r-q)T}$ does not hold, investors can execute strategies that can ex-

ploit index arbitrage. If $F < Se^{(r-q)T}$, an arbitrageur can enter into a long forward contract and short the stock to lock in a riskless profit. If $F > Se^{(r-q)T}$, an arbitrageur can buy the stock and enter into a short forward contract to lock in a riskless profit.

If $F < Se^{(r-q)T}$, index arbitrage is often done by a pension fund that owns an indexed portfolio of stocks whilst if $F > Se^{(r-q)T}$, index arbitrage is often done by a corporation holding short-term money market investments.

N.B.: If the dividend varies during the life of the forward contract but is a known function of time, it can be shown that $F = Se^{(r-q)T}$ still holds with q equal to the *average dividend yield rate during the life of the forward contract.*

7. Futures/forward contracts on currencies: Let S be the current price in dollars of one unit of the foreign currency. A foreign currency has the property that the holder of the currency can earn interest at the risk-free interest rate prevailing in the foreign country. The holder, for example, can invest the currency in a foreign-denominated bond or a foreign-denominated savings account. Define r_f as the value of the foreign risk-free rate with continuous compounding. Consider an investor adopting the following strategy: (a) Buy $e^{-r_f T}$ of the foreign currency (b) Short a forward contract on one unit of the foreign currency. The holding in the foreign currency grows to one unit at time T because of the interest earned. Under the terms of the forward contract, this is exchanged for F at time T. The strategy, therefore, leads to an initial outflow of $Se^{-r_f T}$ and final inflow of F. The PV of the inflow must equal the outflow. Therefore, $Se^{-r_f T} = Fe^{-rT}$ or $F = S^{(r-r_f)T}$, which is the well-known interest-rate parity relationship from the field of international finance. A foreign currency is analogous to a security paying a known dividend yield. The "dividend yield", q, is the risk-free rate of interest in the foreign currency, r_f. Hence,

$$F = S^{(r-r_f)T}.$$

The above is the well-known interest-rate parity relationship from the field of international finance.

8. Futures contracts on the following currencies are trading in the IMM: Japanese yen, Canadian dollar, British pound and Australian dollar. In the case of Japanese yen, prices are expressed as the number of cents per unit of foreign currency. In the case of other currencies, prices are quoted as the number of US dollars per unit of foreign currency.

   Note that spot and forward rates are expressed as the number of units of the foreign currency per US dollar. So, a forward quote on the Canadian dollar of 1.200 would become a futures quote of 0.8333.

9. It is important to distinguish between commodities that are held solely for investment (e.g., gold and silver) and those that are held primarily for consumption.

10. For investment commodities, if storage costs are zero, they can be regarded as being analogous to securities paying no income. Note that storage costs can be viewed as negative income. So, if S is the current spot price of the commodity, r is the risk-free rate, T is the time to maturity, U is the present value of all storage costs that will be incurred during the life of a futures contract then $F = (S + U)e^{rT}$. If u is the storage costs per annum as a proportion of the spot price then $F = Se^{(r+u)T}$. If these relations do not hold, an investor can carry out strategies that would take advantage of arbitrage opportunities.

    Arbitrage arguments can be used to obtain exact futures prices in the case of investment commodities. However, **we showed that they can only be used to give an upper bound to the futures price in the case of consumption commodities.**