Statistical Sciences 4521G/9521B
Assignment No.2
Due Date: 04 March 2014, before the start of lecture at WSC 248

NOTE: Only problems marked with $\star$ are required for submission. The other problems are intended as additional practice exercises. For this problem set, the required problems are numbers 20, 21 and 22.

GUIDELINES ON SUBMITTING ASSIGNMENTS

- Do not submit your rough work! Do the problem set and then re-write it at least once - neatly, with adequate amount of clear explanation. The rewriting stage is the most important one for finding errors in one's work, and it will also deepen your understanding of the subject matter. ***Assignments are marked for both technical correctness and elegance of presentation.***

- Bear in mind to include a sufficient amount of explanation about your work so that any marker does not have to guess what you mean. The grader of your work will determine if you understand what you are writing, not merely that you reach the particular correct answer.

- On questions where a computer output is required, include the output in the text of your answer at the appropriate locations - do not put it all in a bunch at the end of your assignment. Unless, you are instructed to submit your work in a CD or disc, you are expected to hand in a PRINTED COPY.

- YOU MUST WRITE YOUR OWN WORK IN YOUR OWN WORDS, using full sentences and proper English grammar. It is your responsibility to familiarise yourself with the provisions of the University Regulation concerning academic integrity and honesty. Any behaviour that can potentially lead to plagiarism and cheating is a serious offence and carries with it severe penalty.

Do as indicated. ENJOY!

1. Suppose the Black-Scholes price c_t of a call option on a non-dividend paying stock is given by

$$c_t = S_t\Phi(d_1) - Ke^{-r(T-t)}\Phi(d_2)$$

where $d_1 = \dfrac{\ln(S_t/K) + (r + \sigma^2/2)(T - t)}{\sigma\sqrt{T - t}}$ and $d_2 = d_1 - \sigma\sqrt{T - t}$. Show that the hedging parameter theta is given by $\Theta = -rKe^{-r(T-t)}\Phi(d_2) - S_t\Phi'(d_1)\dfrac{\sigma}{2\sqrt{T - t}}$.

2. Explain how a stop-loss hedging scheme can be implemented for the writer of an out-of-the-money option. Why does it provide a relatively poor hedge?

3. What does it mean to assert that the theta of an option position is -0.1 when the time is measured in years?

4. The Black-Scholes price of an out-of-the-money call option with an exercise price of \$40 is \$4. A trader who has written the option plans to use a stop-loss strategy. The trader's plan is to buy at \$40.10 and to sell at \$39.90. Estimate the expected number of times the stock will be bought or sold.

5. "The procedure for creating an option position synthetically is the reverse of the procedure for hedging the option position." Explain this statement.

6. Why did portfolio insurance not work well on 19 October 1987?

7. A fund manager has a well-diversified portfolio that mirrors the performance of the S&P500 and is worth \$360 million. The value of the S&P500 is 1200 and the portfolio manager would like to buy insurance

against reduction of more than 5% in the value of the portfolio over the next six months. The risk-free rate is 6% per annum. The dividend yield on both the portfolio and the S&P is 3% and the volatility of the index is 30% per annum.

(i) If the fund manager buys traded European put options, how much would the insurance cost?

(ii) Explain carefully alternative strategies open to fund manager involving traded European call options, and show that they lead to the same result.

(iii) If the fund manager decides to provide insurance by keeping part of the portfolio risk-free securities, what should the initial position be?

8. A bank's position in options on the dollar-euro exchange rate has a delta of 30,000 and a gamma of -80,000. Explain how these numbers can be interpreted. The exchange rate (dollars per euro) is 0.90. What position would you take to make the position delta-neutral? After a short period of time, the exchange rate moves to 0.93. Estimate the new delta. What additional trade is necessary to keep the position delta-neutral? Assuming the bank did set up a delta-neutral position originally, has it gained or lost money from the exchange rate movement?

9. If Π is the value of a portfolio, show that the various terms of the "Greeks" satisfy the relation

$$\Theta + rS\Delta + \frac{1}{2}\sigma^2 S^2 \Gamma = r\Pi$$

for:

(a) A single European call option on a non-dividend-paying stock
(b) A single European put option on a non-dividend-paying stock
(c) Any portfolio of European put and call options on a non-dividend-paying stock.

10. Explain carefully why a distribution with a heavier left tail and less heavy right tail than the lognormal distribution gives rise to a downward sloping volatility.

11. The market price of a European call is $3.00 and its Black-Scholes model with volatility of 30% is $3.50. The price given by this Black-

Scholes model for a European put option with the same strike price and time to maturity is $1.00. What should the market price of the put option be? Explain the reasons for your answer.

12. Explain what is meant by crashopobia.

13. A stock price is currently $20. Tomorrow, news is expected to be announced that will either increase the price by $5 or decrease the price by $5. What are the problems in using Black-Scholes to value one-month options on the stock?

14. What volatility smile is likely to be observed for six-month options when the volatility is uncertain and positively correlated to the stock price?

15. What pattern in the plot of implied volatility is likely to be caused by jumps in the underlying asset price? Is the pattern likely to be more pronounced for a 2-year option than for a 3-month option?

16. Suppose that a central bank's policy is to allow an exchange rate to fluctuate between 0.97 and 1.03. What pattern of implied volatilities for options on the exchange rate would you expect to see?

17. What are the major problems in testing a stock option pricing model empirically?

18. Option traders refer to deep-out-of-the-money options as being options on volatility. Why do you think they do this?

19. "The Black-Scholes model is used by traders as an interpolation tool." Discuss this view.

20. ⋆ Assume the notation used in the Black-Scholes option pricing formula. Prove the claim $S\Phi'(d_1) = Xe^{-r(T-t)}\Phi'(d_2)$ given in the lecture, which was employed to derive the Δ of a European call. [4 points]

21. ⋆ [Problems 18.25 and 18.26 of Hull, 8th ed] A financial institution has the following portfolio of over-the-counter options on sterling (GBP):

Type	Position	Δ of option	Γ of option	ϑ of option
Call	$-1,000$	0.50	2.2	1.8
Call	-500	0.80	0.6	0.2
Put	$-2,000$	-0.40	1.3	0.7
Call	-500	0.70	1.8	1.4

A traded option A is available with a $\Delta_A = 0.6$, $\Gamma_A = 1.5$ and $\vartheta_A = 0.8$. A traded option B is also available with $\Delta_B = 0.1$, $\Gamma_B = 0.5$ and $\vartheta_B = 0.6$. How could the portfolio be made simultaneously delta-, gamma- and vega-neutral? [8 points]

22. ⋆ [Problem 19.21, Hull, 8th ed]
A futures price is currently \$40. The risk-free interest rate is 5%. Some news is expected tomorrow that will cause the volatility over the next three months to be either 10% or 30%. There is a 60% chance of the first outcome and a 40% chance of the second outcome. Calculate (i.e., construct) a volatility smile for three-month options. Make sure you include in your solution the numerical algorithm/procedure (not code), at least in outline form. [8 points]

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