SS 4521G/FM 9521B
Assignment No.3

Due Date: 11:30am, Thursday, 03 April 2014

Put your assignment in the drop box, labelled SS 4521G/FM 9521B, located across WSC 240.

Note: The assigned TA will monitor arrival of assignments, and as per the course outline UNDER NO CIRCUMSTANCES WILL LATE ASSIGNMENTS BE ACCEPTED.

NOTE: Only problems marked with $\star$ are required for submission. The other problems are intended as additional practice exercises. For this problem set, the required problems can be found on pages 5 and 6.

GUIDELINES ON SUBMITTING ASSIGNMENTS

- Do not submit your rough work! Do the problem set and then re-write it at least once - neatly, with adequate amount of clear explanation. The rewriting stage is the most important one for finding errors in one's work, and it will also deepen your understanding of the subject matter. ***Assignments are marked for both technical correctness and elegance of presentation.***

- Bear in mind to include a sufficient amount of explanation about your work so that any marker does not have to guess what you mean. The grader of your work will determine if you understand what you are writing, not merely that you reach the particular correct answer.

- On questions where a computer output is required, include the output in the text of your answer at the appropriate locations - do not put it all in a bunch at the end of your assignment. Unless, you are instructed to submit your work in a CD or disc, you are expected to hand in a PRINTED COPY.

- YOU MUST WRITE YOUR OWN WORK IN YOUR OWN WORDS, using full sentences and proper English grammar. It is your responsibility to familiarise yourself with the provisions of the University Regulation concerning academic integrity and honesty. Any behaviour that can potentially lead to plagiarism and cheating is a serious offence and carries with it severe penalty.

Do as indicated. ENJOY!

ESTIMATING VOLATILITIES

1. (Problem 22.2) What is the difference between the exponentially weighted moving average model and the GARCH (1,1) model for updating volatilities?

2. (Problem 22.3) The most recent estimate of the daily volatility of an asset is 1.5% and the price the asset at the close of trading yesterday was \$30.00. The parameter λ in the EWMA model is 0.94. Suppose that the price of the asset at the close of trading today is \$30.50. How will this cause the volatility to be updated by the EWMA model?

3. (Problem 22.6) A company uses the GARCH (1,1) model for updating volatility. The three parameters are ω, α and β. Describe the impact of making a small increase in each of the parameters whilst keeping the others fixed.

4. (Problem 22.7) The most recent estimate of the daily volatility in the USD-GBP exchange rate is 0.6% and the exchange rate at 4pm yesterday was 1.5000. The parameter λ in the EWMA model is 0.9. Suppose that the exchange rate at 4pm today proves to be 1.4950. How would the estimate of the daily volatility be updated?

5. (Problem 22.22.8) Assume that S&P500 at close of trading yesterday was 1040 and the daily volatility of the index was estimated as 1% per day at that time. The parameters in a GARCH (1,1) model are $\omega = 0.000002$, $\alpha = 0.06$ and $\beta = 0.92$. If the level of the index at close of trading today is 1060, what is the new volatility estimate?

6. (Problem 22.11) Suppose the current daily volatilities of asset X and asset Y are 1.0% and 1.2%, respectively. The prices of the assets at close of trading yesterday were \$30 and \$50 and the estimate of the

coefficient of correlation between the returns on the two assets made made at this time was 0.50. Correlations and volatilities are updated using a GARCH (1,1) model. The estimates of the model's parameters are $\alpha = 0.04$ and $\beta = 0.94$. For the correlation $\omega = 0.000001$, and for the volatilities $\omega = 0.000003$. If the prices of the two assets at close of trading today are \$31 and \$51, how is the correlation estimate updated?

AMERICAN OPTIONS

7. (Problem 14.15) Consider an American call option on a stock. The stock price is \$70, the time to maturity is eight months, the risk-free rate of interest is 10% per annum, the exercise price is \$65, and the volatility is 32%. A dividend of \$1 is expected after three months and again after six months. Show that it can never be optimal to exercise the option on either of the two dividend dates.

8. (Problem 14.21) Consider an American call option on a stock. The stock price is \$50, the time to maturity is 15 months, the risk-free rate of interest is 8% per annum, the exercise price is \$55, and the volatility is 25%. Dividends of \$1.50 are expected in 4 months and 10 months. Show that it can never be optimal to exercise the option on either of the two dividend dates. Calculate the price of the option.

EXOTIC DERIVATIVES

9. (Problem 25.1) Explain the difference between a forward start option and a chooser option.

10. (Problem 25.2) Describe the payoff from a portfolio consisting of a lookback call and a lookback put with the same maturity.

11. (Problem 25.7) Explain why a down-and-out put is worth zero when the barrier is greater than the strike price.

INTEREST-RATE DERIVATIVES

12. (Problem 13.9) It has been suggested that the short-term interest rate,r_t, follows the stochastic process

$$dr_t = a(b - r_t)dt + r_t c dW_t$$

where a, b, and c are positive constants and W_t is a standard Brownian motion. Describe the nature of this process.

13. (Problem 13.11) Suppose that x_t is the yield to maturity with continuous compounding on a zero-coupon bond that pays off \$1 at time T. Assume that x_t follows the process

$$dx_t = a(x_0 - x_t)dt + s x_t dW_t$$

where a, x_0, and s are positive constants and W_t is a standard Brownian motion. What is the process followed by the bond price?

PROBLEMS REQUIRED FOR SUBMISSION

14. ($\star$) (Problems 22.17-22.18) **[9 points]**
Suppose the price of gold at close of trading yesterday was \$600 and its volatility was estimated as 1.3% per day. The price at the close of trading today is \$596. Update the volatility estimate of the gold price using
(a) The EWMA model with $\lambda = 0.94$ [2.5 pts]
(b) The GARCH (1,1) model with $\omega = 0.000002$, $\alpha = 0.04$ and $\beta = 0.94$. [2.5 pts]
(c) Now, suppose that in addition to the information given above, the price of silver at the close of trading yesterday was \$16, its volatility was estimated as 1.5% per day, and its correlation with gold was estimated as 0.8. The price of silver at the close of trading today is unchanged at \$16. **Update the volatility of silver price and the correlation between silver and gold prices** using the two models under consideration in (a) and (b). In practice, is the ω parameter likely to be the same for gold and silver? [4 pts]

15. ($\star$) (Not in Hull) **[5 points]**
What is the value in Canadian dollars of a derivative that pays off 1,000,000 euros in 6 months provided that the Canadian dollar/euro exchange rate is greater than 1.40 at that time? Suppose the current exchange rate is 1.36 (i.e., \$ 1.36 CAD = 1 euro). Assume that the Canadian dollar and euro interest rates are 3% and 5% per annum, respectively. The volatility estimate of the exchange rate is 15% per annum.

16. ($\star$) (Problem 25.32) **[6 points]**
Outperformance certificates (also called "sprint certificates, "accelerator certificates, or "speeders) are offered to investors by many European banks as a way of investing in a companys stock. The initial investment equals the companys stock price, S_0. If the stock price goes up between time 0 and time T, the investor gains k times the increase at time T where k is a constant greater than 1.0. However, the stock price used to calculate the gain at time T is capped at some maximum level M. If the stock price goes down the investors loss is equal to the decrease. The investor does not receive dividends.
(a) Show that the outperformance certificate is a package. [3 pts]
(b) Calculate the price of a one-year outperformance certificate when the stock price is 50 euros, $k = 1.5$, $M = 70$ euros, the risk-free rate is 5%, and the stock price volatility is 25%. Dividends equal to 0.5 euros are expected in 2 months, 5 month, 8 months, and 11 months. [3 pts]

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