Statistical Sciences 4521G/FM 9521B

SOLUTIONS TO REQUIRED PROBLEMS
Assignment No. 3

(☆) Question #14 [9 points]

2.5 (a) The proportional change in the price of gold is $-4/600 = -0.00667$. Using the EWMA model the variance is updated to

$$0.94 \times 0.013^2 + 0.06 \times 0.00667^2 = 0.00016153$$ 2.0

so that the new daily volatility is $\sqrt{0.00016153} = 0.01271$ or 1.271% per day. 0.5

2.5 (b) Using GARCH (1,1) the variance is updated to

$$0.000002 + 0.94 \times 0.013^2 + 0.04 \times 0.00667^2 = 0.00016264$$ 2.0

so that the new daily volatility is $\sqrt{0.00016264} = 0.1275$ or 1.275% per day. 0.5

1.5 (c) EWMA 4

The proportional change in the price of silver is zero. Using the EWMA model the variance is updated to

$$0.94 \times 0.015^2 + 0.06 \times 0 = 0.0002115$$

so that the new daily volatility is $\sqrt{0.0002115} = 0.01454$ or 1.454% per day. 0.5

The initial covariance is $0.8 \times 0.013 \times 0.015 = 0.000156$. For the EWMA, the covariance is updated to 0.5

$$0.94 \times 0.000156 + 0.06 \times 0 = 0.00014664$$

so that the new correlation is $0.00014664/(0.01454 \times 0.01271) = 0.7934$ 0.5

2 GARCH (1,1)

Using GARCH (1,1) the variance is updated to

$$0.000002 + 0.94 \times 0.015^2 + 0.04 \times 0 = 0.0002135$$

so that the new daily volatility is $\sqrt{0.0002135} = 0.01461$ or 1.461% per day.

The initial covariance is $0.8 \times 0.013 \times 0.015 = 0.000156$. So, for the GARCH (1,1) the covariance is updated to

$$0.000002 + 0.94 \times 0.000156 + 0.04 \times 0 = 0.00014864$$

so that the new correlation is $0.00014864/(0.01461 \times 0.01275) = 0.7977$.

Context. Reason: ω must be different for different markets. (0.25)

For a given α and β, the ω parameter defines the long run average value of a variance or a covariance. There is no reason why we (0.25) should expect the long run average daily variance for gold and silver should be the same. There is also no reason why we should expect the long run average covariance between gold and silver to be the same as the long run average variance of gold or the long run average variance of silver. ***In practice, therefore, we are likely to want to allow ω in a GARCH(1,1) model to vary from market variable to market variable.***

(☆) Question #15 [5 points]

It is instructive to consider two different ways of valuing this instrument.

I. (2) From the perspective of a European investor it is a cash-or-nothing put. The variables are S_0 = 1/1.36 = 0.73529, X = 1/1.4 = 0.71429, r = 0.05 q = 0.03 σ = 0.15, T = ½.

The derivative pays off if the exchange rate is less than 0.71429. The value of the derivative is 1,000,000$\Phi(-d_2)$ $e^{-0.05(1/2)}$ where

(2)

$$d_2 = \frac{\ln\left(\frac{0.73529}{0.71429}\right) + (0.05 - 0.03 - 0.15^2/2)}{0.15\sqrt{1/2}} = \frac{0.028976 + 0.00875}{0.15\sqrt{1/2}} = 0.355684$$

Since $\Phi(-d_2)$ = 0.361039, the value of the derivative is 1,000,000(0.361039)$e^{-0.05(1/2)}$ (0.5) or 352,124.92 euros. In dollars this is 352,124.92 (1.4) = **$492,974.88.** (0.5)

II. (2) [Alternative solution] From the perspective of a dollar investor the derivative is an asset-or-nothing call. The variables are S_0 = 1.36 = 0.73529, X = 1.4, r = 0.03 q = 0.05 σ = 0.15, T = ½. The value is 1,000,000$\Phi(d_1)$ $e^{-0.05(1/2)}$ where

(2)

$$d_1 = \frac{\ln\left(\frac{1.36}{1.40}\right) + (0.03 - 0.05 + 0.15^2/2)}{0.15\sqrt{1/2}} = \frac{-0.028987537 - 0.00875}{0.106066017} = -0.35579$$

Since $\Phi(d_1)$ = 0.360999, the value of the derivative is 1,000,000(0.360999)$e^{-0.05(1/2)}$ (0.5) or 352,085.92 euros. In dollars this is 352,085.90 (1.4) = **$492,920.26.** This is almost the same as the result in (I). The slight difference is due to rounding errors in the calculation. (0.5)

(✫) Question #16 [6 points]

a) The outperformance certificate is equivalent to providing a return on an initial investment equal to the stock price consisting of

T

(i) A long position in k one-year European call options on the stock with a strike price equal to the current stock price.

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(ii) A short position in k one-year European call options on the stock with a strike price equal to M

1 each.

(iii) A short position in one European one-year put option on the stock with a strike price equal to the current stock price.

T = 1 year in (6)

b) In this case the present value (using the Black-Scholes formula) of the three parts to the gain are

1 (i) $1.5 \times 5.0056 = 7.5084$

1 (ii) $-1.5 \times 0.6339 = -0.9509$

0.75 (iii) -4.5138

The total of these is $7.5084 - 0.9509 - 4.5138 = 2.0437$ 0.25

Remark: The present value of the return of the initial investment is $50e^{-0.05\times1}$=47.56. The total present values of what will be received is therefore 49.6. This is less than the initial investment of 50.