SS4521G - 24–28 February 2014

SUMMARY OF IMPORTANT POINTS DISCUSSED IN THE LECTURE

The following concepts/theories were covered/reviewed:

1. Portfolio managers sometimes employ the creation of put options synthetically in order to insure a portfolio of stocks (equities). The strategy involves either (i) by trading the portfolio or (ii) by trading index futures on the portfolio. In the first case (i.e., trading portfolio) the portfolio is split between equities and risk-free assets (e.g., money market accounts or government securities). When market declines, more of the investment is allocated in the risk-free securities whilst when the market moves up, more of the investment is allocated in the equities. equities. For the second case (i.e., trading index futures), the equity portfolio is kept intact and index futures are sold. As market declines, more index futures are sold whilst as the market moves up fewer are sold. Such strategies of creating put options synthetically will work well under normal conditions. During a stock market crash (e.g., 19 October 1987, when the Dow Jones dropped more than 500 points), these strategies will not work well. Portfolio managers are unable to sell either stocks or index futures fast enough to protect their positions.

2. Using put-call parity, we showed that the dollar pricing error when the Black-Scholes model is used to price a European put option is the same as the dollar pricing error when it used to price a European call option with the same strike price and time to maturity. This finding shows that the implied volatility of a European call option is always the same as the implied volatility of a European put option when the two have the same strike price and maturity. So, when traders refer to the relationship between implied volatility and strike price, or to relationship between implied volatility and maturity, there is no need to specify whether they are talking about calls or puts.

3. The plot of implied volatility versus the strike for foreign currency options forms a volatility smile. The volatility is relatively low for at-the-money options and it becomes progressively higher as an option moves either in-the-money or out-of-the-money. Consider for example a deep-out-of-the-money call option with a high strike price X_2. It was illustrated in the lecture that the probability that exchange rate being greater than X_2 is higher for the implied probability distribution than for the lognormal distribution. Hence, there is an expectation that the implied distribution will give a relatively high price for the option. Consequently, a relatively high price leads to a relatively high implied volatility. An analogous reasoning goes for a deep-out-of-the-money put option with a low strike price X_1.

4. We also looked at empirical evidence that supports the volatility smile used by traders for foreign currency. In particular, it shows that the lognormal distribution understates the probability of extreme movements in exchange rates.

5. There are two reasons why exchange rates are not lognormally distributed: (i) The volatility of the asset is not constant and (ii) The asset price changes may not be smooth at some instances and have jumps occasionally.

6. The volatility smile used by traders to value equity options (both those on individual stocks and those on stock indices) has the general form/shape called *volatility skew*. The implied volatility decreases as the strike price increases. For this situation, the implied distribution has a heavier left tail and a less heavy right tail than the lognormal distribution.

7. Interestingly, volatility skew is a phenomenon observed for equities that only existed since the stock market crash. Implied volatilities were much less dependent on strike price. Mark Rubinstein (1994), in a paper published in the Journal of Finance, suggested that one reason for this is due to "crashophobia". Traders and investors are concerned about the possibility of another crash similar to October 1987 and they attempt to price options accordingly. In fact, it is usually seen that whenever the market declines, there is a tendency for the skew to become more pronounced.

8. Another possible explanation for the volatility skew in equity options is the so-called leverage. As company's equity declines in value, the company's leverage increases (using debt-to-equity ratio). This means that the equity becomes more risky and its volatility increases. As company's equity increases in value, leverage decreases. The equity then becomes less risky and its volatility decreases. This reasoning shows that we can expect the volatility of equity to be a decreasing function of equity value consistent with the volatility skew.

9. When a single large jump is anticipated in the price of a stock due to an important announcement, the volatility smile becomes a volatility "frown". At-the-money options have higher volatilities than either out-of-the-money or in-the-money options.

10. An example on how to back out implied volatilities for the case when a single large jump in the stock price is expected was also discussed. This involves the mixture of lognormal distributions for the stock price.

11. Traders construct the so-called volatility term structure. The implied volatility of an option then depends on its life. A volatility surface is produced when volatility smiles and volatility term structures are combined. This defines implied volatility as a function of both strike price and the time to maturity. This is used as an interpolating tool to estimate appropriate volatility inputs in pricing options.