SS4521G - 03–07 February 2014

SUMMARY OF IMPORTANT POINTS DISCUSSED IN THE LECTURE

The following concepts/theories were covered/reviewed in the context of stock price processes evolving on a two-period binomial tree model in discrete time:

1. Options on futures contract (or futures options) are also traded on many different exchanges. The put-call parity for European options is given by
$$c_t^F + Ke^{-r(T-t)} = p_t^F + F_t e^{-r(T-t)}$$
where F_t is the futures price at time t and c_t^F and p_t^F are the corresponding values of call and put options on futures price. Assuming that the futures price has a lognormal distribution, the pricing analysis of the futures option is the same as that of an option on a stock with dividend yield at rate q. In this case, we set $q = r$.

2. There are also options on commodities that entail cost of carry at the rate c for storing these commodities and conveneience yield y derived from having holding these assets. Examples of these include metals such as gold, silver, copper and commodities for consumption such as lumber, cotton, coffee, corn, etc. The pricing of these options on commodities is the same as that of an option on a stock with dividend at rate q with q being replaced by $y - c$.

3. **Monte-Carlo (MC) simulation technique applied to the pricing of a European call option.**
Given a stochastic differential equation $dS_t = \mu S_t dt + \sigma S_t dW_t$, its discretised version is $S_{k+1} - S_k = \mu S_k \Delta t_k + \sigma S_k \epsilon \sqrt{\Delta t_k}$, where $\Delta t_k = t_{k+1} - t_k$, $\epsilon \sim N(0,1)$ and the sense of the equality is understood to in

be in law or distribution.

We note that S_{k+1} depends on S_k and S_0 must be initialised. If the stochastic differential equation has a solution, it is advisable to directly discretise the solution as it gives more accurate results. The MC technique in derivative pricing was demonstrated for the option on stock price process following GBM dynamics (i.e., the log of the stock price has a normal distribution). The procedure is applied to the discretised version of the GBM solution and is described below:

a) Assume the Black-Scholes framework. Simulate the stock price sample paths starting from the discretised version of the GBM, i.e.,

$$S_{k+1} \stackrel{LAW}{=} S_k \exp\left[\left(r - \frac{\sigma^2}{2}\right)\Delta t_k + \sigma\epsilon\sqrt{\Delta t_k}\right].$$

After generating N sample paths, one obtains N values for S_T.

b) Calculate the N pay-offs from the derivative. In this case, the payoff is $\max(S_T - X, 0)$.

c) After obtaining N pay-offs from the N sample paths, take their average. That is, compute $\frac{\sum_{i=1}^{N} \max\left(S_T^i - X, 0\right)}{N}$, where S_T^i is the terminal stock price produced by the ith sample path. This gives the expected value $E\left[\max\left(S_T - X, 0\right)\right]$.

d) Finally, using an appropriate risk-free rate for the discount factor, evaluate $e^{-r(T-t)}\frac{\sum_{i=1}^{N} \max\left(S_T^i - X, 0\right)}{N}$.

e) It was emphasised that since the calculated value for the option price is just an estimate, this must be accompanied by a confidence interval.

The above procedure was implemented in EXCEL spreadsheet and discussed in the previous few lectures. Examples dealing with option pricing results in spreadsheets were also posted in the course website.

4. The Heston's stochastic volatility model was introduced. This is a model for the volatility process v_t given by $dv_t = \alpha(\beta - v_t)dt + \gamma\sqrt{v_t}dZ_t$. This provides a mean-reverting non-negative volatility process that can can be discretised, simulated and plugged into the Black-Scholes GBM to obtain the price of an option. The Brownian motion Z_t may be correlated (with correlation ρ) to the Brownian motion W_t that governs the Black-Scholes GBM for the stock price.

   Obtaining correlated normally random samples was described in the lecture using Cholesky's decomposition.

5. Aside from delta hedging, we also considered other hedging dimensions and these include the following parameters:

$$
\begin{aligned}
\vartheta &= \frac{\partial c}{\partial \sigma} = S\sqrt{T-t}\phi(d_1) \\
\Theta &= \frac{\partial c}{\partial t} = -\frac{S\sigma}{2\sqrt{T-t}}\phi(d_1) - rKe^{-r(T-t)}\Phi(d_2) \\
\rho &= \frac{\partial c}{\partial r} = (T-t)Ke^{-r(T-t)}\Phi(d_2) \\
\Gamma &= \frac{\partial^2 c}{\partial S^2} = \frac{\phi(d_1)}{S\sigma\sqrt{T-t}}.
\end{aligned}
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   In the above, $\phi(\cdot) = \Phi'(\cdot)$ and $\Phi(\cdot)$ denote the pdf and cdf of a standard normal RV, respectively.

6. A financial institution that sells an option has a financial obligation (faced with risk) to fulfill in the future. We illustrated in the lecture that the following 3 strategies: (i) naked position (means to do nothing), (ii) covered position (means buying the shares as soon as the option has been sold) and (iii) stop-loss strategy, do not always lead to a satisfactory hedge. A stop-loss strategy involves buying one unit of the stock as soon as its price rises above K and selling it as soon as price falls below K.

Remark: *One can use Monte-Carlo simulation to assess the overall performance of stop-loss strategy. This involves sampling of paths for the stock price and observing the results using the scheme.*

7. We considered delta-hedging. The parameter Δ represents the change of the option price with respect to the price of the underlying asset; this can be viewed as the slope of the curvature representing the price of an option.

8. Since the naked and covered positions, and the stop-loss strategy do not yield satisfactory results, we **need** to employ more sophisticated hedging schemes in order to fulfill financial obligations in the future.