SS4521G - 13–17 January 2014

SUMMARY OF IMPORTANT POINTS DISCUSSED IN THE LECTURE

The following concepts/theories were covered/reviewed in the context of stock price processes evolving on a two-period binomial tree model in discrete time:

1. In the computation of $\int_0^t (dW_s)^2$, we considered the term

$$\lim_{\|\pi_n\| \to 0} \sum_{i=0}^{n-1} \left[W_{t_{i+1}} - W_{t_i} \right]^2,$$

   where $\|\pi_n\|$ is the norm of the partition of $[0, t]$. This is called the quadratic variation of W_t.

2. If we follow the Newton-Leibniz calculus, $dW_t^2 = 2W_t dW_t$. We showed however that in stochastic calculus, this not the case as we would obtain $dW_t^2 = 2W_t dW_t + dt$. In integral form,

$$W_t^2 = 2 \int_0^t W_s dW_s + t.$$

   The second term t is the correction term coming from the quadratic variation. It has to be there so that both sides of the above equality (with integral on the right-hand side) have equal expectations.

   Note that we proved $E\left[\int_0^t W_s dW_s \right] = 0$ by approximating the integral and using the fact that Brownian motion has stationary and independent increments.

3. In developing Ito's lemma, with W_t being a BM, it was mentioned and heuristically explained that we have the "multiplication rule": $(dW_t)^2 = dt$, $(dt)^2 = 0$ and $(dW_t)(dt) = 0$.