SS4521G - 10–14 March 2014

SUMMARY OF IMPORTANT POINTS DISCUSSED IN THE LECTURE

The following concepts/theories were covered/reviewed:

1. When EWMA is used, estimation is simpler since we set $\omega = 0$, $\alpha = 1 - \lambda$ and $\beta = \lambda$. In this case, only one parameter needs to be estimated.

2. We focused our discussion on GARCH (1,1) model. Setting V_L, the target variance rate, to the sample variance calculated from the data or other values believed to be reasonable is a technique proposed by Engle and Merzich and this simplifies the parameter estimation.

3. Model validation: Due to the phenomenon of volatility clustering/persistence, we note that the series $\{u_i^2\}$ exhibits autocorrelation. If a GARCH model is working well, it is should remove the autocorrelation in the series $\left\{\frac{u_i^2}{\sigma_i^2}\right\}$. A scientific way of testing for autocorrelation is through the use of Ljung-Box (LB, 1978) statistic. The LB statistic, H_0, H_a together with the critical values for a 95 % confidence level and a lag of $k = 15$ were given in class.

4. For GARCH (1,1), the best estimate of the variance rate on date $n + k$ is given by $E\left[\sigma_{n+k}^2\right] = V_L + (\alpha + \beta)^k \left[\sigma_n^2 - V_L\right]$. A proof of this result was outlined in the lecture.

5. To obtain a stable GARCH (1,1), one must have $\alpha+\beta<1$. Otherwise, if $\alpha+\beta>1$, γ is negative and the variance rate is "mean fleeing" instead of "mean-reverting".

6. Volatility term structure: Assume that we have an option to value between day n and $n+N$. The expected variance rate during the life of the option is $\frac{1}{N}\sum_{k=0}^{N-1} E\left[\sigma_{k+n}^2\right]$. The square root of this expected variance rate can be used as the volatility estimate appropriate for pricing the $N-$day option under the GARCH (1,1) framework.

7. Correlation calculation is needed in the VaR computation and GARCH (1,1) model can be extended to do this. Let x_i's and y_i's be percentage changes in market variables X_i and Y_i, respectively. That is,
$x_i=\frac{X_i-X_{i-1}}{X_{i-1}}$ and $y_i=\frac{Y_i-Y_{i-1}}{Y_{i-1}}$.
$\sigma_{x,n}$: daily volatility of X at day n
$\sigma_{y,n}$: daily volatility of Y at day n
cov_n : estimated covariance between daily percentage changes in X and Y on day n.

$$\rho_{xy,n}=\frac{\text{cov}_n}{\sigma_{x,n}\sigma_{y,n}},\quad \sigma_{x,n}^2=\frac{1}{m}\sum_{i=1}^m x_{n-i}^2$$

and

$$\sigma_{y,n}^2=\frac{1}{m}\sum_{i=1}^m y_{n-i}^2,\quad \text{cov}_n=\frac{1}{m}\sum_{i=1}^m\sum_{i=1}^m x_{n-i}y_{n-i}.$$

8. For the EWMA model, $\text{cov}_n=\lambda\text{cov}_{n-1}+(1-\lambda)x_{n-1}y_{n-1}$.

9. In order that the variance and covariance are calculated consistently, the semi-positive definite condition must be satisfied. That is, for any

$n \times 1$ vector $\mathbf{w}$ and $n \times n$ covariance matrix $\mathbf{\Omega}$ we must have $\mathbf{w}^\top \mathbf{\Omega} \mathbf{w} > 0$, where $\top$ denotes the transpose of a vector.

10. We discussed and showed the result that when there are dividends, it can only be optimal to exercise at a time immediately before the stock goes ex-dividend.

    In particular, we assume that n ex-dividend dates are anticipated and that they are at times $t_1,\ t_2,\ ,\ldots,\ t_n$, with $t_1 < t_2 < \ldots < t_n$. The dividends corresponding to these times will be denoted by $D_1,\ D_2,\ ,\ldots,\ D_n$, respectively.

    We showed that if $D_{n-1} \leq X[1 - e^{-(t_n - t_{n-1})r}]$, it is not optimal to exercise immediately prior to time t_{n-1}. Similarly, for $i < n$, if

    $$D_i \leq X[1 - e^{-r(t_{i+1}) - r(t_i)}],$$

    it is not optimal to exercise immediately prior to time t_i.