SS4521G - 10–14 February 2014

SUMMARY OF IMPORTANT POINTS DISCUSSED IN THE LECTURE

The following concepts/theories were covered/reviewed:

1. If an investor, for example, has sold call option contracts, i.e., options to buy x number of shares then his position can be hedged by buying Δ times x number of shares. The gain (loss) on the option position would tend to offset the loss (gain) on the stock position.

2. The delta of the stock by definition is $+1$. The delta of the asset position offsets the delta of the option position. A position with a delta of zero is called delta-neutral.

3. The gamma of an option is sometimes referred to as its <u>curvature</u>. If gamma is small, this means that delta changes only slowly and adjustments to keep a portfolio delta-neutral need only be made infrequently. If the gamma is large in absolute terms, delta is highly sensitive to the price of the underlying asset. For instance, (refer to the illustration given in class) when the stock price moves from S to S', delta assumes that the option price moves from c to c' when in fact it moves from c to c''. The difference between c' and c'' leads to a hedging error. The error depends on the curvature of the relationship between the option price and the stock price. Gamma measures this curvature. Variations of gamma with time to maturity were shown in the lecture.

4. The parameter theta is almost always negative for a call option. This is because as time to maturity decreases the option tends to become less valuable. Theta is sometimes referred to as the <u>time decay</u> of the

portfolio. Unlike delta, it does not really make sense to look at theta as a hedging tool since the passage of time is certain. But *theta could serve as a proxy for gamma in a delta-neutral portfolio.* Using Taylor's series (or Itô's lemma), once can show that for a delta-neutral portfolio Π,

$$\delta\Pi = \theta\ (\delta t) + \frac{1}{2}\Gamma(\delta S)^2.$$

Variations of theta with time to maturity were shown in the lecture.

5. An important question we considered is how to make a delta-neutral portfolio also gamma-neutral. Assume that we have a delta-neutral portfolio with gamma Γ and a traded option with gamma Γ_T is available. Then, we need to add $\omega_T = -\dfrac{\Gamma}{\Gamma_T}$ of traded options to the portfolio to make the portfolio gamma-neutral. However, the position in the underlying asset has to be changed to maintain delta-neutrality. If one wishes to hedge an option against the movement of the volatility, he needs to consider a vega-neutral position. This will involve taking an offsetting position in a traded option. In general, if a trader wants to obtain a portfolio that is both gamma- and vega-neutral, two traded options are necessary.

   Numerical examples were provided in the lecture demonstrating how to achieve a portfolio that is simultaneously gamma-, vega- and delta-neutral.

6. In addition to monitoring the "Greeks" of the portfolio, practitioners also perform the so-called ***scenario analysis***. This type of analysis examines the effects of possible alternative future movements in market variables on the value of a portfolio.

7. We discussed volatility smiles in an attempt to answer the following questions: (i) How close are option prices from the market to those

given by the Black-Scholes formula? (ii) Do traders really use the Black-Scholes model in pricing options? (iii) Are the probability distributions of asset prices really lognormal?

8. We considered the concept of implied volatility. This is the volatility implied from an option price in the market using the Black-Scholes model or a similar model. In other words, when implied volatility is plugged into the Black-Scholes model we obtain the market option price. As the Black-Scholes formula cannot be inverted to find an expression for the volatility in terms of other parameters, the implied volatility calculation requires numerical methods (e.g., Newton-Raphson technique as presented in the lecture).