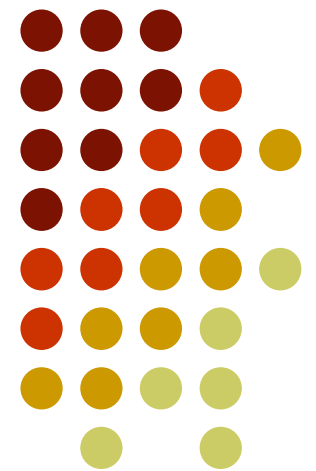


Exotic Options



Types of Exotics



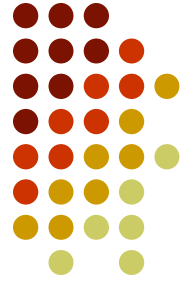
- Package
- Nonstandard American options
- Forward start options
- Compound options
- Chooser options
- Barrier options
- Binary options
- Lookback options
- Shout options
- Asian options
- Options to exchange one asset for another
- Options involving several assets
- Other recent innovations

Packages



- Portfolios of standard options
 - Examples from **AS 2557 & SS 3520**: bull spreads, bear spreads, straddles, etc
- Often structured to have zero cost
 - One popular package is a range-forward contract (long call + short put OR short call and long put)

Non-Standard American Options



- Exercisable only on specific dates (Bermudans)
- Early exercise allowed during only part of life (initial “lock out” period)
- Strike price changes over the life (warrants, convertibles)

Forward Start Options



- Option starts at a future time, T_1
- Most common in employee stock option plans
- Often structured so that strike price equals asset price at time T_1



Compound Option

- Option to buy or sell an option
 - Call on call
 - Put on call
 - Call on put
 - Put on put
- Can be valued analytically
- Price is quite low compared with a regular option

Chooser Option “As You Like It”



- Option starts at time 0, matures at T_2
- At T_1 ($0 < T_1 < T_2$) buyer chooses whether it is a put or call
- This is a package!



Chooser Option as a Package

At time T_1 the value is $\max(c, p)$

From put-call parity, $p = c + e^{-r(T_2-T_1)}X - S_1e^{-qT_2-T_1}$.

The value at time T_1 is therefore

$$\begin{aligned}\max(c, p) &= \max(c, c + e^{-r(T_2-T_1)}X - S_1e^{-qT_2-T_1}) \\ &= c + e^{-qT_2-T_1} \max(0, Xe^{-(r-q)(T_2-T_1)} - S_1).\end{aligned}$$

This is a call with strike price X plus $e^{-qT_2-T_1}$ put options with strike $Xe^{-(r-q)(T_2-T_1)}$.

Barrier Options



- Option comes into existence only if stock price hits barrier before option maturity
 - ‘In’ options (*knock-in*)
- Option dies if stock price hits barrier before option maturity
 - ‘Out’ options (*knock-out*)



Barrier Options (continued)

- Stock price must hit barrier from below
 - 'Up' options
- Stock price must hit barrier from above
 - 'Down' options
- Option may be a put or a call
- Eight possible combinations



Parity Relations

$$c = c_{ui} + c_{uo}$$

$$c = c_{di} + c_{do}$$

$$p = p_{ui} + p_{uo}$$

$$p = p_{di} + p_{do}$$



Binary or Digital Options

- Cash-or-nothing: pays A if $S_T > X$, otherwise pays nothing.
 - Value = $e^{-rT} A N(d_2)$
- Asset-or-nothing: pays S_T if $S_T > X$, otherwise pays nothing.
 - Value = $S_0 N(d_1)$
- Options on the performance of a sports team or level of snow in a certain area

Decomposition of a Call Option



Long Asset-or-Nothing option

Short Cash-or-Nothing option where payoff
is X

$$\text{Value} = S_0 N(d_1) - e^{-rT} X N(d_2)$$



Lookback Options

- Lookback call pays $S_T - S_{\min}$ at time T
 - Allows buyer to buy stock at lowest observed price in some interval of time
- Lookback put pays $S_{\max} - S_T$ at time T
 - Allows buyer to sell stock at highest observed price in some interval of time
- Analytic solution

Shout Options



- Buyer can ‘shout’ once during option life
- Final payoff is either
 - Usual option payoff, $\max(S_T - X, 0)$, or
 - Intrinsic value at time of shout, $S_\tau - X$
- Payoff: $\max(S_T - S_\tau, 0) + S_\tau - X$
- Similar to lookback option but cheaper
- A binomial tree be used to value a shout option.



Asian Options

- Payoff related to average stock price
- Average Price options pay:
 - Call: $\max(S_{\text{ave}} - X, 0)$
 - Put: $\max(X - S_{\text{ave}}, 0)$
- Average Strike options pay:
 - Call: $\max(S_T - S_{\text{ave}}, 0)$
 - Put: $\max(S_{\text{ave}} - S_T, 0)$



Asian Options

- No analytic solution
- Can be valued by assuming (as an approximation) that the average stock price is lognormally distributed
- Can be European or American.



Exchange Options

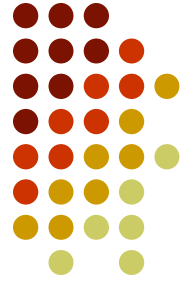
- Option to exchange one asset for another
- For example, an option to exchange one unit of U for one unit of V
- Payoff is $\max(V_T - U_T, 0)$

Basket Options



- A basket option is an option to buy or sell a portfolio of assets
- This can be valued by calculating the first two moments of the value of the basket and then assuming it is lognormal

Interest rate derivatives



- Bond options
- Caps [portfolio of caplets]
- Floors [portfolio of floorlets]
- Captions
- Floortions
- Swaptions, deferred swaps

Other innovations

- Parisian options-crossover between barrier and Asian options
- Passport or vacation options-options on the balance of a trading account (one can also have barrier or chooser passport options)
- Israeli options-options give the *option seller* the ability to cancel the option early - at the expense of a payment to the holder of the option whilst maintaining the early-exercise feature for the holder of the option.





Russian option- an option where a minimum payout m to the buyer is guaranteed. The payout is given as the discounted maximum price that the option has ever traded at during the life of the option and can be extremely beneficial for the option holder. In a sense, this option is a perpetual American-style lookback option.

Possibilities in Exotic options are virtually limitless.



How Difficult is it to Hedge Exotic Options?

- In some cases exotic options are easier to hedge than the corresponding vanilla options. (e.g., Asian options)
- In other cases they are more difficult to hedge (e.g., barrier options)

Static Options Replication



- This involves approximately replicating an exotic option with a portfolio of vanilla options
- Underlying principle: if we match the value of an exotic option on some boundary, we have matched it at all interior points of the boundary
- Static options replication can be contrasted with dynamic options replication where we have to trade continuously to match the option