SS 4521G/FM 9521B - Advanced Financial Modelling
Outline of Lectures: 13-17 January 2014

For this week, we aim to cover the following theories/concepts:

Recap of last week's lectures

1. Random walk model

2. Brownian motion W_t

Calculus of Brownian motion

3. Analysing the qualitative behaviour of W_t, its properties and how it's being employed in financial modelling

4. Stochastic differential equation of the form $dX_t = \mu(X_t, t)dt + \sigma(X_t, t)dW_t$

5. Itô's lemma/Itô's differentiation rule

6. The "multiplication rule"

7. The geometric Brownian motion as a model for price dynamics

8. Some applications and examples

9. Simulation of random walk and its convergence to Brownian motion

10. Partial differential equation (PDE) approach in valuation: The intent here is to derive the PDE, with terminal condition, that satisfies the price of a derivative security. Here, we shall consider a portfolio whose value is Π in which the holder is short one derivative security and long an amount of $\frac{\partial f}{\partial S}$ of shares, where S is the price of the underlying asset and f is the price of a derivative security.

11. The Feynman-Kac's result linking the solution of a PDE to conditional expectation will be presented without proof

12. Girsanov theorem on change of probability measures: We shall examine the conditions that permit the change from physical/objective to risk-neutral/martingale measure.

13. The stock price dynamics under the risk-neutral measure will be determined.

14. Risk-neutral pricing of European options: The Black-Scholes option pricing formula will be derived using the risk-neutral approach. The put-call parity will be revisited.

15. We shall look at how the Black-Scholes-Merton European call price gets modified to take into account when (i) the underlying asset pays a known dividend, (ii) the underlying is a stock index, (iii) the underlying is a currency, (iv) the underlying is a commodity price and (v) the underlying is a futures contract.