Theorem. Transformation Theorem for Double Integrals. *If*

(i) *T is a one-to-one transformation between open sets Ω in the uv-plane and $\Psi = T(\Omega)$ in the xy-plane, defined by continuously differentiable functions $x = x(u, v)$ and $y = y(u, v)$;*

(ii) *the Jacobian $J(u, v) = \dfrac{\partial(x, y)}{\partial(u, v)}$ is nonzero throughout Ω;*

(iii) *R and S are bounded sets in the xy-plane and uv-plane, respectively, whose closures are subsets of Ψ and Ω, respectively ($\overline{R} \subset \Psi, \overline{S} \subset \Omega$), such that $R = T(S)$;*

(iv) *$f(x, y)$ is defined over R and $g(u, v) \equiv f(x(u, v), y(u, v))$;*

then

(v) *R has area if and only if S has area;*

(vi) *assuming R and S have area, f is integrable over R if and only if $g|J|$ is integrable over S, and in case of integrability*

(5) $$\iint_R f(x, y)\, dA = \iint_S g(u, v)\, |J(u, v)|\, dA.$$

(Cf. Fig. 1315.)

A similar theorem holds for multiple integrals of any number of variables.

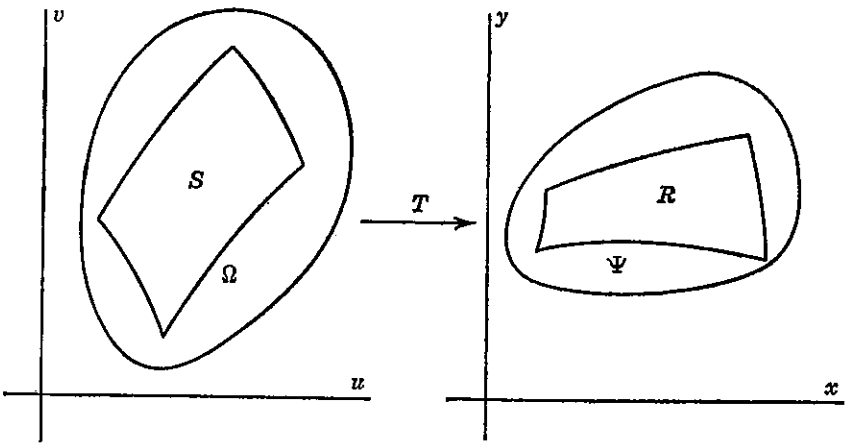

FIG. 1315